PRELIMINARY ANALYSIS FOR LUNAR ROVING VEHICLE STUDY GROUND DATA SYSTEMS AND OPERATIONS

JPL CONTRACT NO. 952668

VOLUME III OF III

ROVING VEHICLE NAVIGATION

(ELEVATION DETERMINATION ANALYSES)

HUGHES

HUGHES AIRCRAFT COMPANY
SPACE SYSTEMS DIVISION

30 JUNE 1970

HUGHES REFERENCE NO. C0077

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SUMMARY

The study contract for preliminary analysis for Lunar Roving Vehicle (LRV) Ground Data Systems (GDS) and Operations was initiated in September 1969, and was originally intended as a six-month task. During February 1970, Hughes was directed to stretch out the existing program with a Final Report delivery scheduled for 30 June 1970.

The content of the Final Report (Volumes I, II, and III) constitutes only part of the final product by the Hughes Aircraft Company in response to Contract No. 952668. The total response is listed below and it should be noted that material printed in Jet Propulsion Laboratory documents was supplied under the terms of the contract in final draft form.

- Preliminary Analysis for Lunar Roving Vehicle Study – Ground Data Systems and Operations, Hughes Reference No. C0077, dated 30 June 1970: Volume I, Roving Vehicle Guidance (Remote Driving Study); Volume II, Roving Vehicle Payload (Science Mode Time Analyses); Volume III, Roving Vehicle Navigation (Evaluation Determination Analyses).

- LRV Navigation and Guidance System Phase A Study Report, JPL Document No. 760-42, dated 15 October 1969: Section V, Mission Operations; Section VI, Navigation and Guidance Operations; Section VII, Problem Areas; Section X, Plan for Phase B Study.

- Science Ground Data System and Science Operations Organization for Remotely Controlled Lunar Traverses – Phase A Study Report, JPL Document No. 760-39, dated 10 October 1969: Section VI, Science Operations; Section VII, Problem Areas, Section X, Phase B Study Plan.

- Operations Profiles for Lunar Roving Missions, JPL Document No. 760-46, dated May 1970.

Originally, it was not planned for Hughes to participate in Phase A Report preparation. The basic responsibility of Hughes in the early part of this contract had been to assist JPL in defining all mission-dependent Earth activities and resources (hereinafter referred to as the "Mission Operations Complex" – MOC) required to support the remote-controlled phase of the

LRV Mission. Since this constituted a variable which is dependent upon mission requirements, the total plan which implemented these requirements was first established.

Mission requirements also dictate a general LRV design. Such a design is not necessary in defining a general MOC but becomes necessary in establishing its details (commensurate to the extent of available LRV design detail). No sole LRV design existed throughout the contract period. Bendix and Grumman each had several designs in the early part of the period and JPL therefore postulated a single design to act as a baseline for the Hughes effort of MOC definition. Considerable time was spent coordinating with JPL sources regarding establishing and periodically up-dating a postulated LRV design without incompatibilities and with a level of detail useful toward MOC definition. Continuing assistance to JPL was provided in assessing the effect on the GDS baseline design of the LRV mission, vehicle, and science payload changes during the study, and design change recommendations were made as appropriate.

It was originally intended to deliver to JPL detailed definition of the Ground Data System in the areas of display, operations profile, operations organization, navigation programs and computer applications, hazard prediction programs, and avoidance maneuver techniques. During January and February of 1970, it was determined by JPL that the study should concentrate more in the areas of 1) remote driving problems, 2) Navigational analyses for operations use (concentrating on elevation determinations), and 3) time line analyses. In particular, it was decided to develop the above definitions to only the intermediate level and not initiate work on a general command and control computer program, or identify a single operations organization. It has also been intended to expand the detail of the MOC to a level of detail attainable within the remainder of the contract period. However, this effort was also suspended at JPL's request. Thus, during January and February of 1970, a report entitled "MOC Definition for Synthesized LRV Design" was submitted. This report consisted of five basic sections plus an appendix; and included an Introduction, Synthesized LRV Description, Operations Profile, and MOC Profiles. This material was used by JPL in preparing the Phase B Report.

The MOC profile charts in the Phase B Report show the direct correlation of all the particular Earth-based activities and equipment used to implement each specific operational activity identified by numerical subdivision of a basic operation "mode" (the first divisional level within the remote controlled phase of the mission). The estimated Delta time to accomplish each row of the MOC charts was also calculated. An iteration with specific operational activities and general mission plans is required to establish total mission time lines. This was not pursued further in the areas of Guidance and Navigation by Hughes at JPL's request.

Paralleling the above in time was an effort by Hughes to identify (for operational use) the subtle aspects of perhaps the most demanding of the LRV mission requirements--Navigation and Guidance. A review was made of all available documentation produced by Bendix, General Motors, and Grumman regarding the subject. Preliminary investigation in Navigation by landmark showed that accuracy versus number of visual landmarks, and accuracy versus number of navigation updates for a given course, was not a simple relationship. Subsequent investigations established appreciably reliable criteria for operational decisions when navigating by landmark.

Volume I details considerations applicable to aid remote driving by superimposing driving aids on the TV panorama. These aids are used by the Remote Driver at the Remote Controller Position while the vehicle is in motion. The vehicle general design baseline is first established.

Volume II contains four detailed time line studies of portions of the Stationary Science Mode. These studies provide an additional link in the continuing iterative process of defining the LRV mission operations procedure, ground equipment, and administrative organization.

Volume III is mainly concerned with elevation determination. Some early unfinished work on Rover Navigation is also presented. Preliminary error curves of Rover position as affected by landmark orientation with respect to LRV path are shown; also, a table representing a partial comparison of various navigation schemes is included. The elevation determination methods considered are 1) use of the basic LRV instruments, 2) addition of a ranging Laser and precision inclinometer, 3) tracking an orbiter from the Rover, and 4) miscellaneous techniques including a stable platform, one or more star trackers, a sun seeker, gyrocompassing, Foucauld pendulum, and differential ranging. The intent of the volume is to provide sufficient information concerning a variety of navigation and elevation determination methods to permit filtering out of less attractive schemes.

LUNAR NAVIGATION

GENERAL

Preliminary functional requirements for LRV navigation were reviewed by Hughes Aircraft Company. Hughes-recommended Ground Data System Functional Navigational Requirements and Operations Organization were included in the formal issue of Jet Propulsion Laboratory Document Number 760-42 entitled "LRV Navigation and Guidance Systems Phase A Study Report" dated 15 October 1969, Sections V and VI.

The Phase B Rover Navigation Study was formally initiated in early December 1969, and represents approximately three (3) man-months of effort. Feasible types of navigational computer programs were considered based on assumed baseline on-board instrumentation as identified in JPL Document No. 760-46 dated May 1970, "Operations Profiles for Lunar Roving Missions". The use of computers in providing operational aids for navigation by landmark recognition from image, laser range, and other on-board data was also considered and recommendations made.

Prior to initiation of the lunar elevation determination study, the navigation of an LRV was considered. Although it was originally intended to provide a specific choice for LRV navigation on the lunar surface, Jet Propulsion Laboratory later defined the intended tradeoffs and their criteria of accuracy, time, and limitation to be beyond the scope of this effort and instead directed that the study was to be concentrated more in the area of lunar altitude determination. Specifically, vehicle position determination in selenographic coordinates was not to be attempted.

INTRODUCTION

Many aspects of the navigation and elevation determination problems are similar in that much of the potential instrumentation can be applied to both. The two problems should be solved as a general navigation system.

Although the present study was concerned mainly with elevation determination, enough work was done on the horizontal navigation to realize what is needed for that part of the system. Use of landmark navigation for the horizontal case clearly seems the way to go. Therefore, the choice of methods for elevation determination should be slanted toward use of the local lurain unless an all-around better method can be demonstrated. Results of the study should be viewed in this perspective.

The LRV navigation work involved, in a very preliminary way, four aspects: (a) a brief literature search; (b) a review of lunar navigation computer programs; (c) a start on a landmark error analysis; and (d) preliminary comparison of navigation methods. Although these efforts were abandoned at the direction of JPL in order to make the elevation determination study, the preliminary results are summarized here.

LITERATURE REVIEW

The purpose of the literature search was to acquire background on: (a) current requirements of lunar science missions; (b) landmark navigation; (c) gravity gradiometry and gravitation measurements; (d) lunar gravitational anomalies; (e) modern methods for precision measurements; and (f) lasers.

References 1 through 6 satisfied requirements (a) and (b). References 9 through 12 and 14 were used for (c). References 15 through 19 covered item (d) adequately. Vendor specifications on various types of equipment and miscellaneous sources such as Reference 23, private communications, etc. provided data on such matters as (for example) methods for measuring a displacement to

10Å accuracy. Finally, information on laser characteristics and laser uses for ranging and holography was obtained from Hughes Research Labs at Malibu and from Reference 20.

LUNAR NAVIGATION COMPUTER PROGRAMS

Four basic lunar navigation methods have been computer programmed and studied. They are: passive; nongyro; celestial-inertial; RF ranging; and landmark navigation.

The first three techniques have been studied extensively by Bendix under contract (Reference 1) to NASA. The Bendix study consisted of a computerized error analysis including the errors associated with initial position fix, dead-reckoning and piloting. The programs also accommodate ranging, doppler and angle reference information inputs from lunar orbiting satellites. The computer program is conveniently modularized.

Inputs to the program consist of all conceivable error sources afflicting the particular navigation method. These include errors in the appropriate navigation instruments, ephemeris errors, local vertical anomalies, computation errors, wheel slippage and altitude variations.

The output is position error in selenographic coordinates as functions of the input errors. Position error ellipsoids can be constructed from the output.

The program is comprehensive and rather elaborate, involving some 280 primary equations. Errors in the various navigation systems are summarized in Reference 25.

The apparent complexity is due to the inclusion of nearly the whole gamut of navigation instruments as alternatives. However, the modularization feature should permit real-time use of the program for a particular vehicle. This is because only those modules in the program concerned with the necessary instrumentation for a particular navigation system need be utilized. For example, it should be feasible to combine the initial fix and dead reckoning modules of that program with an independently developed program for landmark navigation to obtain vehicle position uncertainty due to all causes.

The fourth method, landmark navigation, was the subject of the Hughes Study. It was programmed for the GE 635 computer. The objectives of the study were expected to show vehicle position uncertainty as related to (1) orientation of landmarks re vehicle path, (2) number of fixes on a particular landmark, (3) number of landmarks, (4) optimum step length, and (5) initial fix, odometer and bearing angle errors.

LRV NAVIGATION PROCEDURE

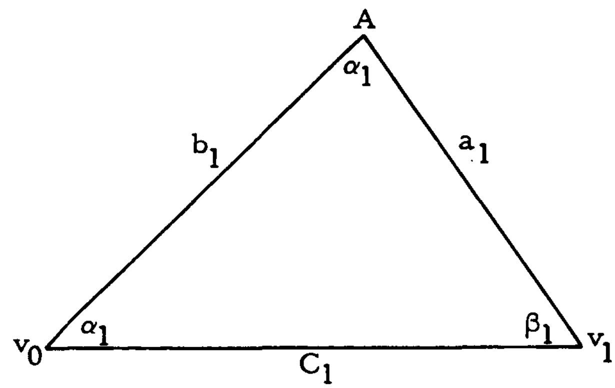
Because of the JPL experiments on landmark navigation in the local desert and their interest in this scheme when combined with dead reckoning as a primary navigation updating method, a brief look was taken at the technique.

The principal conclusion reached was that navigation accuracy depends heavily on position (orientation and distance) of the landmark with respect to the vehicle path.

Referring to the Fig. in Case 1, the LRV travels from a known position v_0 along the path v_0v_1 with a stop at v_1. Angle orientations of landmark A with respect to the desired vehicle path are measured at v_0 and v_1. Distance C_1 is assumed to be obtained from dead reckoning, based on odometer and directional gyro information. Dead reckoning probable errors on C_1 can be obtained from the Bendix computer program. It is assumed that landmark distances are initially unknown.

Case 1. Single Landmark, One Step

Solving triangle v_0v_1A using measured angles α_1 and β_1 ($\pm\Delta\alpha_1$, $\pm\Delta\beta_1$) and measured distance $C_i \pm \Delta C_i$,



$$a_1 = \frac{C_1 \sin \alpha_1}{\sin \gamma_1} \tag{1}$$

$$b_1 = \frac{C_1 \sin \beta_1}{\sin \gamma_1} \; . \tag{2}$$

The RSS errors in a_1 and b_1 are

$$\Delta a_1 \sin \gamma_1 = \left[(\sin \alpha_1 \Delta C)^2 + (C_1 \cos \alpha_1 \Delta\alpha_1)^2 + (C_1 \sin \alpha_1 \cot \gamma_1 \Delta\gamma_1)^2\right]^{\frac{1}{2}} . \tag{3}$$

$$\Delta b_1 \sin \gamma_1 = \left[(\Delta C \sin \beta_1)^2 + (C_1 \cos \beta_1 \Delta\beta_1)^2 + (C_1 \sin \beta_1 \cot \gamma_1 \Delta\gamma_1)^2\right]^{\frac{1}{2}} \tag{4}$$

where

$$\gamma_1 = \pi - \alpha_1 - \beta_1 \text{ and } \Delta\gamma_1 = -\Delta\alpha_1 - \Delta\beta_1 .$$

But since $\Delta\alpha_1$ and $\Delta\beta_1$ are independent measurements,

$$\Delta\gamma_1 = \left(\overline{\Delta\alpha}_1^{\,2} + \overline{\Delta\beta}_1^{\,2}\right)^{\frac{1}{2}} \; . \tag{5}$$

If $\Delta\alpha_1$ includes only the actual errors of angular measurement and not the integrated azimuth (or average heading) error during the step, then the error ellipse at the end of Step 1 is given by Δa_1 and by the odometer error, assuming exact knowledge of v_0 position.

Landmark A position with respect to the LRV is then known from Δb_1 and $b\Delta\alpha_1$ (where α_1 does not include heading error).

The curves of Figures 1 through 4 were obtained from solutions of Eqs. (1) through (4) for the case of one landmark and one step. They indicate:

(a) Initial bearing angle should be small.

(b) The second bearing angle should be greater than 90°.

(c) Position error varies fairly linearly with step length.

(d) Optimum $\gamma = 90^{\circ}$.

The cases of two or more landmarks, two or more steps etc., were not studied.

Tentative conclusions from the curves are: (1) landmarks should be chosen to fall within an angle α_1 of $\pm 60^{\circ}$ to the desired path of the LRV, and (2) it is desirable to pass by the landmark, i.e., maintain the average bearing as nearly a-beam as possible. Additional comments are:

1. Landmark navigation generally appears capable of restricting dead reckoning errors.

2. Depending on angular errors, and on position of landmarks, large odometer errors could actually be reduced. Likewise lateral errors can be reduced to values comparable with in-track errors.

3. Accuracy of landmark navigation may be limited by an as yet undetermined distribution of suitable landmarks with respect to the vehicle track.

4. An attractive feature of landmark navigation is that the necessary elementary computations can easily be performed on the vehicle. The only inputs to the computer would be odometer reading and TV pointing angle with respect to a reference direction.

To judge feasibility of landmark navigation, the following work needs to be done.

(a) Use Equations of type (3) and (4) to limit landmark orientations, as specified by α and β, to those providing errors comparable with odometer errors. The limits will be functions of measurement errors.

(b) Include in (a) the special case of γ near 0° and 180°.

(c) Include in (a) the effect of approximate initial knowledge of distance to landmarks (from maps).

(d) The accuracy with which a TV camera can be pointed toward landmarks is open to question because of varying appearance of certain types of landmarks as viewed from different positions. Some effort needs to be devoted to this question.

(e) Optimize the frequency of updating position.

(f) In order to choose between the numerous navigation methods, a rather detailed block diagram of each method including amplifiers, multipliers, etc. will be required. This is necessary to estimate relative complexity, reliability, telemetering requirements and operational problems.

The final result could be in the form of a matrix in which all of the navigation methods can be compared from any desired point of view such as accuracy, sensitivity to failures, cost, weight, need for redundancy, etc.

Comparison of Navigation Schemes

A comparison of navigation schemes was started, the idea being to show the instruments necessary for each navigation method so that comparisons could be made conveniently from many points of view, such as accuracy, complexity, operations, etc. (see Table I).

In the Table, an "X" mark means that a particular instrument is required for the indicated navigation scheme. An "0" mark means the instrument is not needed. The "or" under sun sensor means either a sun sensor or an earth tracker can be used.

The Table does not exhause the variety of navigation schemes.

Figure 5 shows distance to the lunar horizon a function of altitude. The curves are included for quick reference because of the widespread tendency to underestimate distance to the lunar horizon.

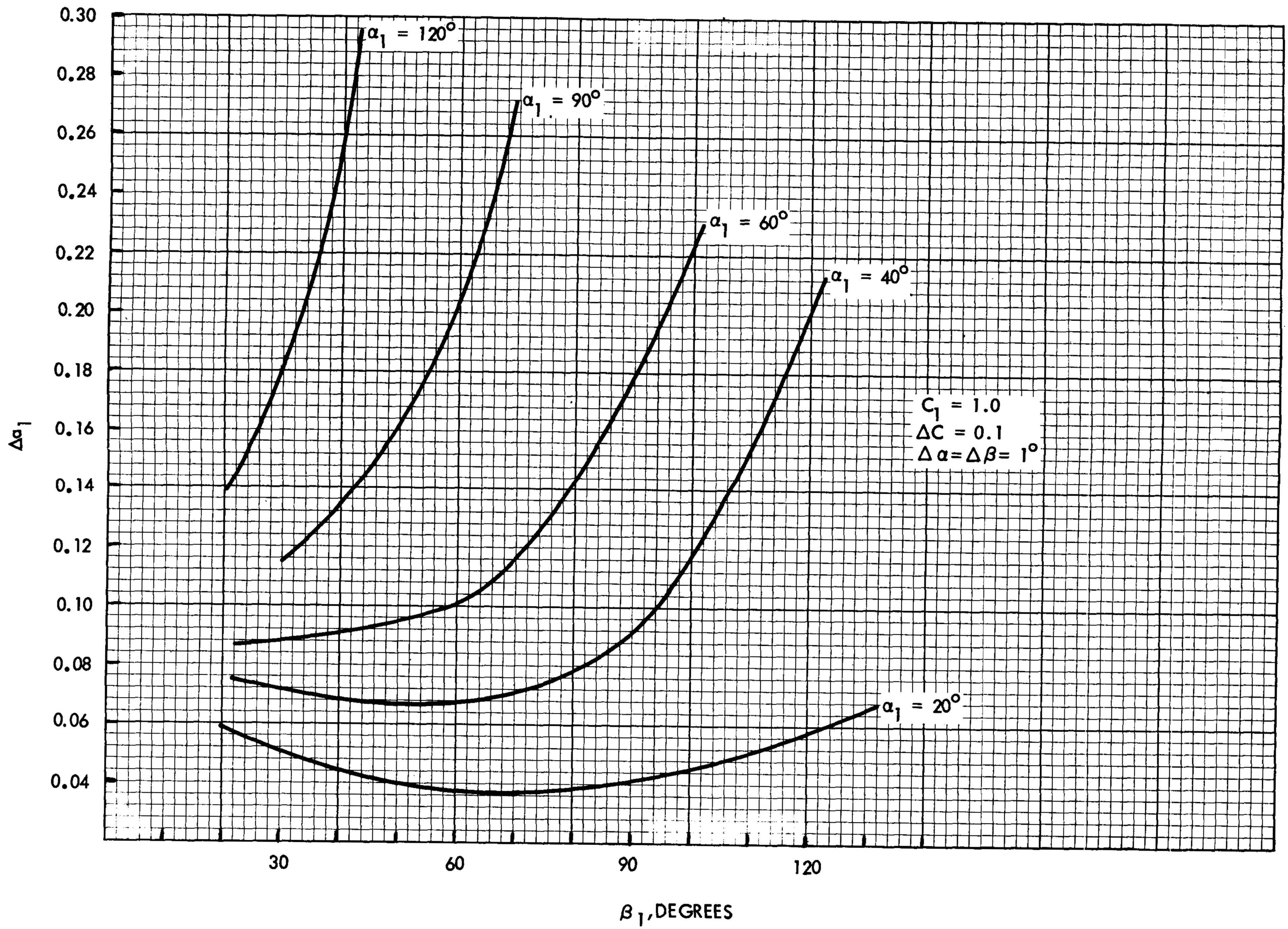


FIGURE 1. POSITION ERROR VERSUS BEARING ANGLE

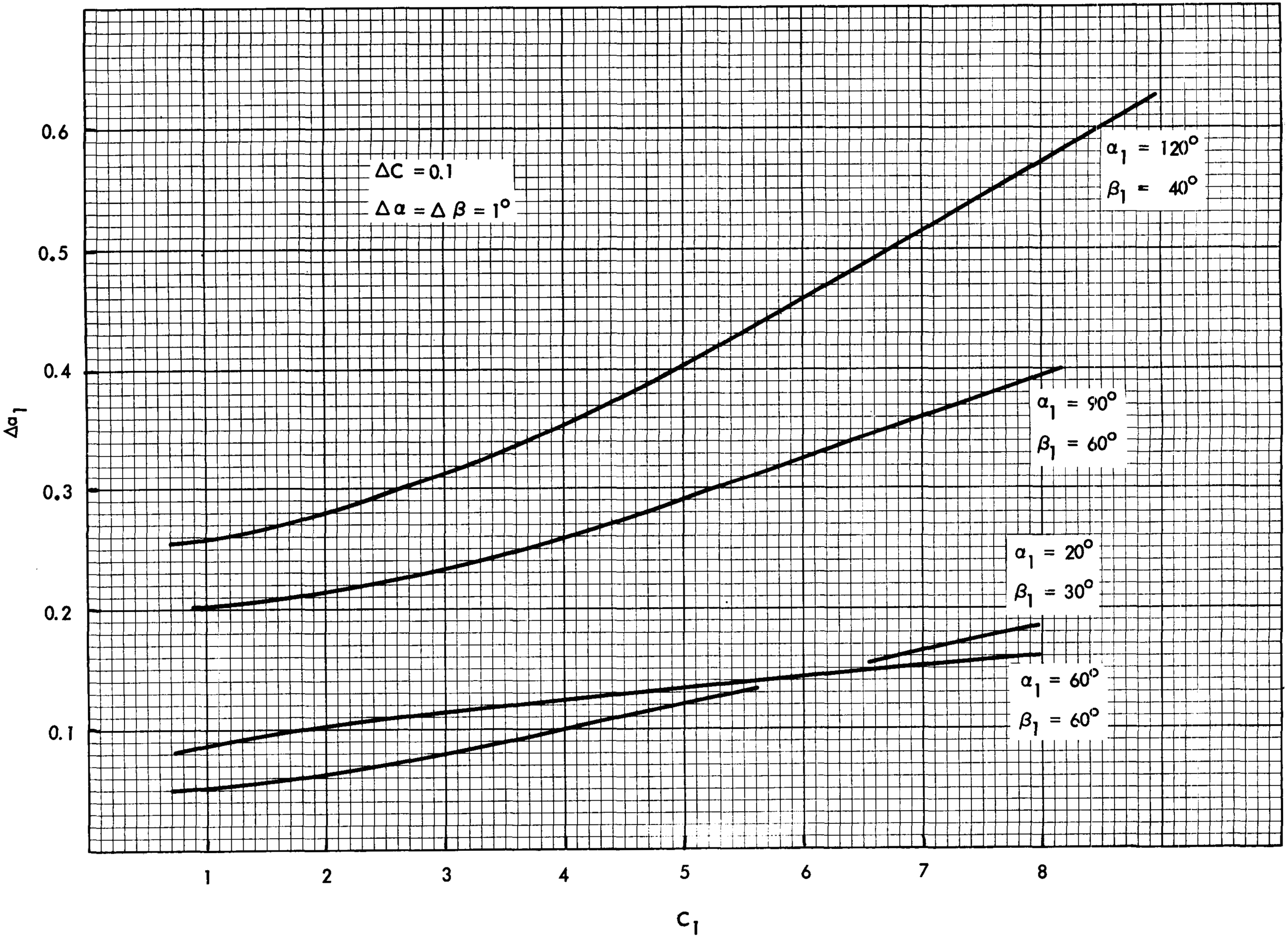


FIGURE 2. POSITION ERROR VERSUS STEP LENGTH

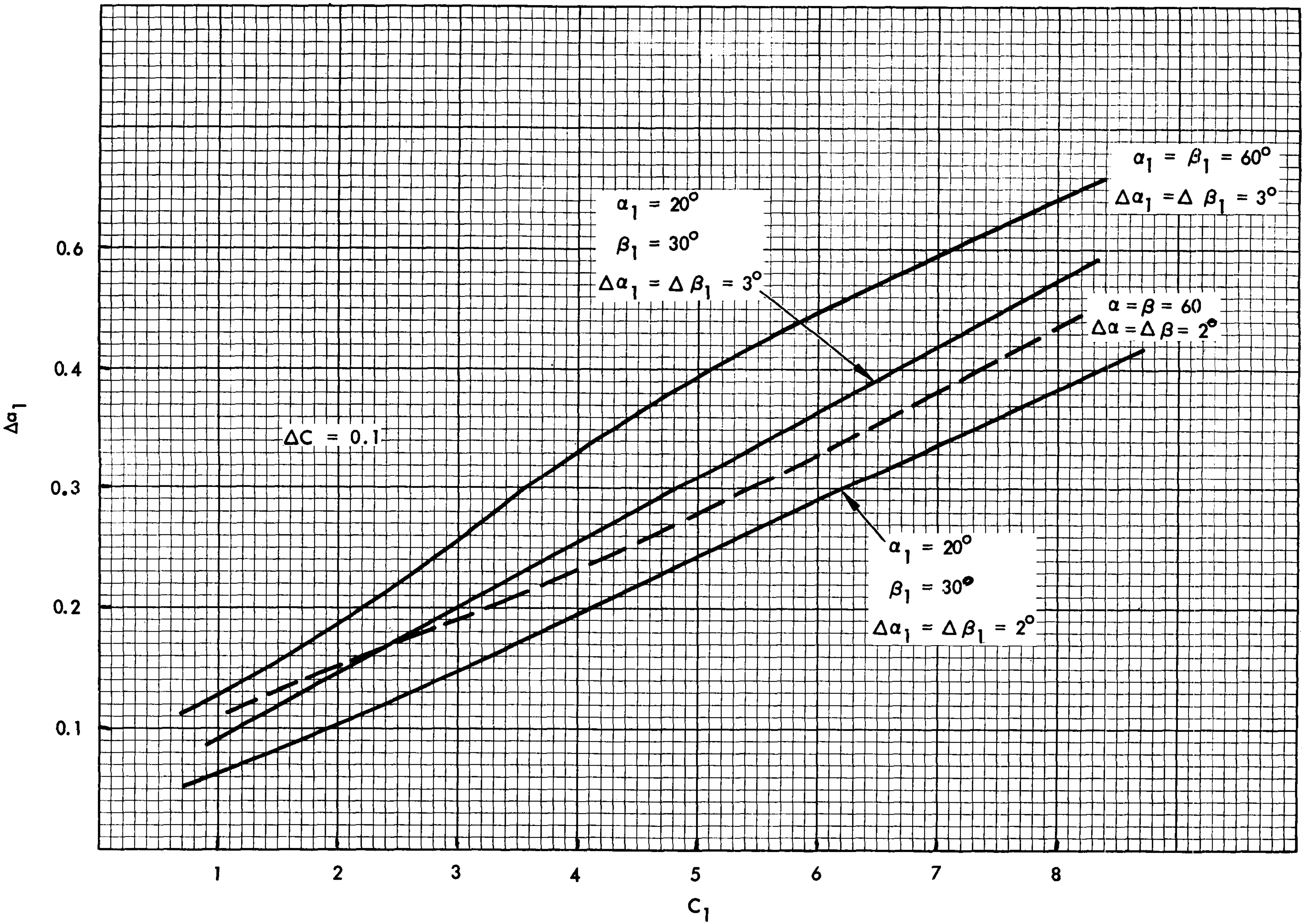


FIGURE 3. POSITION ERROR VERSUS STEP LENGTH

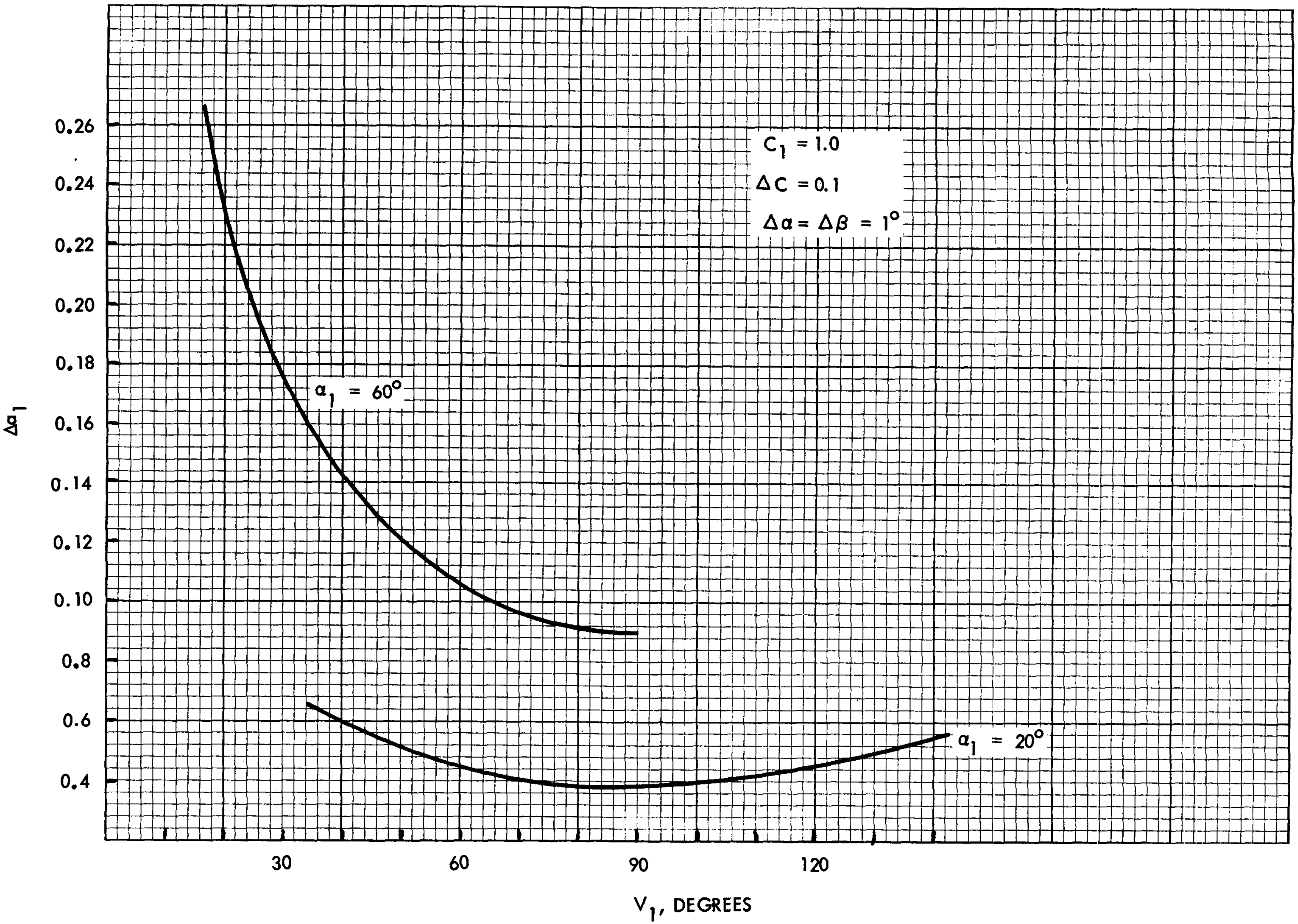


FIGURE 4. POSITION ERROR VERSUS SUBTENDED ANGLE AT LANDMARK

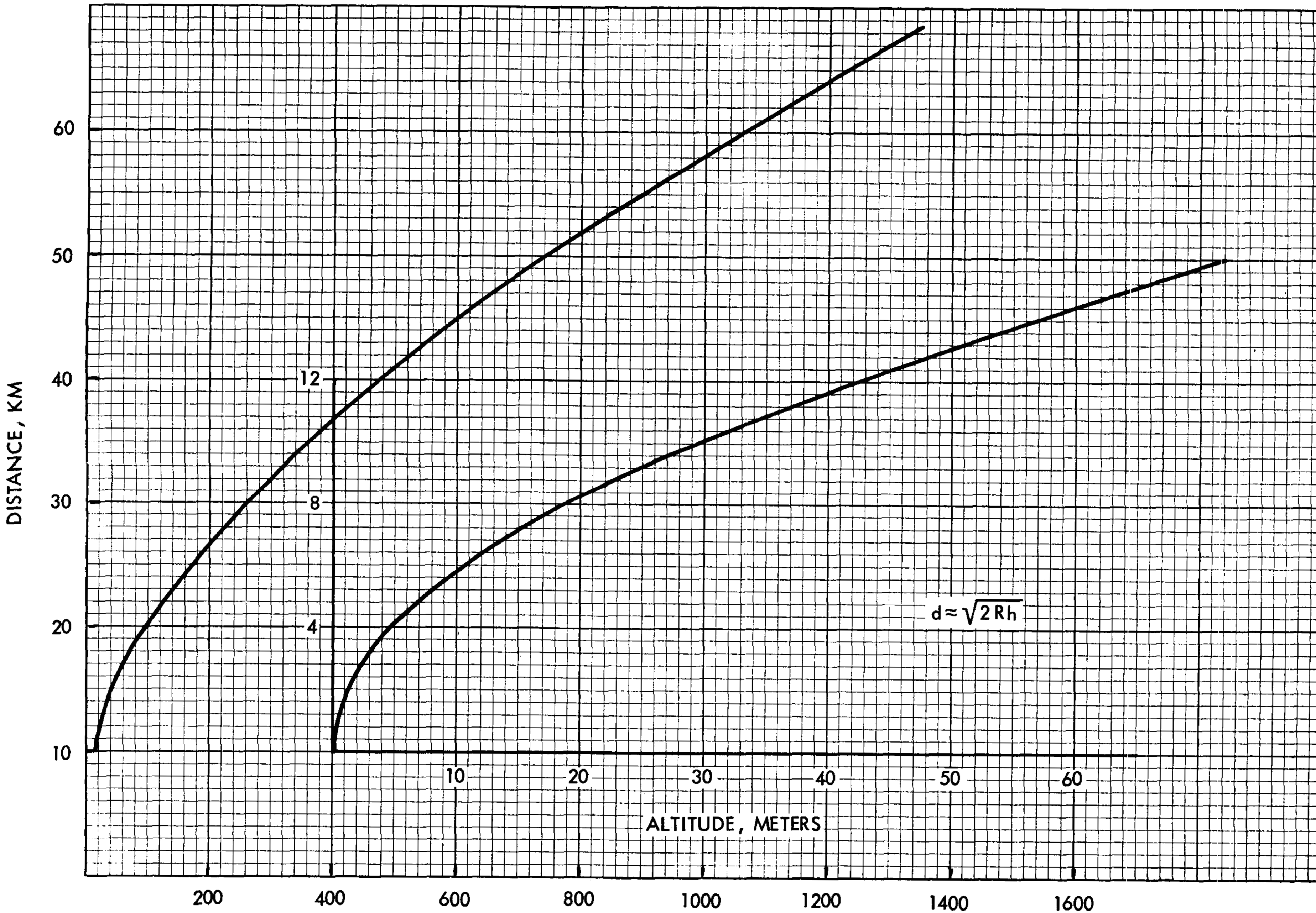


FIGURE 5. LUNAR ALTITUDE VERSUS DISTANCE TO HORIZON

METHOD	INCLINOMETER	ODOMETER	TV	GYRO	SUN SEEKER ASPECT 5^o-65^o	SUN SEEKER ASPECT 65^o-365^o	EARTH SEEKER ASPECT 5^o-65^o	EARTH SEEKER ASPECT 65^o-365^o	LASER	STAR TRACKER	STAB. PLAT.	RELATIVE ERROR FOR TRAVERSE OF (KM) 30	RELATIVE ERROR FOR TRAVERSE OF (KM) 100	RELATIVE ERROR FOR TRAVERSE OF (KM) 1000	TIME FOR FIX	DATA RATES	RELATIVE COMPLEXITY
DEAD RECKONING	X	X	X	0	X or	0	X	0	*	0	0	*	*	*	*	*	*
	X	X	X	X	*	0	*	0	*	X	0	*	*	*	*	*	*
LANDMARK	X	X	X	0	X or	0	X	0	*	0	0	*	*	*	*	*	*
	X	X	X	X	*	0	*	0	*	X	0	*	*	*	*	*	*
CELESTIAL INERTIAL	X	X	X	X	X	0	*	0	*	X	*	*	*	*	*	*	*
	X	X	X	0	*	0	*	0	*	X	X	*	*	*	*	*	*
RF TECHNIQUES	X	X	X	0	*	0	*	0	*	X	*	*	*	*	*	*	*

An **X** means that a particular instrument is required for the indicated navigation scheme.

An 0 means the instrument is not needed.

An * means time did not permit evaluation.

"Aspect" means elevation angle of sun or earth.

Dashed lines attached to the Methods means two alternative instrument arrangements are included.

"OR" means either a sun sensor or an earth tracker can be used.

TABLE I. COMPARISON OF NAVIGATION METHODS

LUNAR ELEVATION DETERMINATION

GENERAL

The purpose of this part of the Lunar Rover study is to devise methods for determination of lunar elevation and to roughly outline their implementation.

Simply stated, the elevation determination problem consists of matching state-of-the-art instrumentation with relative and absolute elevation accuracy requirements as established by geologists. Elevation accuracy requirements will be influenced greatly by cost and reliability of the equipment. The problem has been divided into two parts -- relative and absolute elevation determination.

The accuracy of two basic relative elevation determination methods has been calculated in a form such that tradeoffs can be made between instrument accuracy and elevation accuracy. One method utilizes merely those instruments needed on the Rover for other purposes; i.e., a TV or facsimile camera, a simple inclinometer and odometer, and a relatively inaccurate celestial sensing instrument.

The second method includes a precision inclinometer and an 0.1% ranging device such as a laser or doppler radar. This method is capable of propagating a benchmark altitude a distance of 30 Km with an accuracy considerably better than 10 meters (see Figure 8).

Practical matters may preclude achievement of significantly better accuracies, e.g., a 3 $\widehat{\text{sec}}$. inclinometer accuracy implies that one side or end of the vehicle must not settle into the sand more than 0.001 inch during a measurement.

RELATIVE ELEVATION

Relative elevation is the difference in elevation between a location and some arbitrary benchmark reference location. A Lunar Rover can propagate this

this benchmark elevation by distance measurement, and by use of the local corrected horizontal. With sufficiently accurate instrumentation, certain gravitational anomalies can be detected by use of this method.

Many methods are available for measuring relative elevation. Usually the time required to make a measurement, inaccuracies of the necessary instrumentation, operational complexity, and cost increase rapidly with the required precision.

A variety of methods is outlined below, with expected accuracy and a brief description of each. Actually, much of the instrumentation used in measuring relative elevation is either necessary or useful for navigation of the Rover. Instrumentation for accurate relative elevation determination can be used to provide comparably accurate relative (and in some methods, absolute) position determination.

<u>Method No. 1. Basic LRV Instruments</u>

Presumably the Rover will be equipped with a TV or FAX Camera, a 2-axis inclinometer, an odometer, a celestial sighting device and, for low latitude expeditions, a directional gyro.

With these instruments, relative elevation can be obtained by noting the elevation angle, θ, of an observable as viewed by the camera referenced to the inclinometer. Range, r, to the observable is also necessary and can be obtained from the odometer either by direct measurement or by triangulation in a horizontal plane using the azimuth attitude reference (e.g. sun seeker or gyro). The error in relative altitude is

$$\overline{\Delta h}^2 = \overline{r\Delta\theta}^2 + \overline{\theta\Delta r}^2 \qquad (1)$$

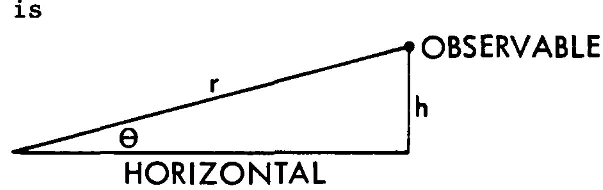


Figure 6 is a plot of the above relation. The uncertainty in θ is caused by camera nonlinearity and image centering, and by inclinometer error

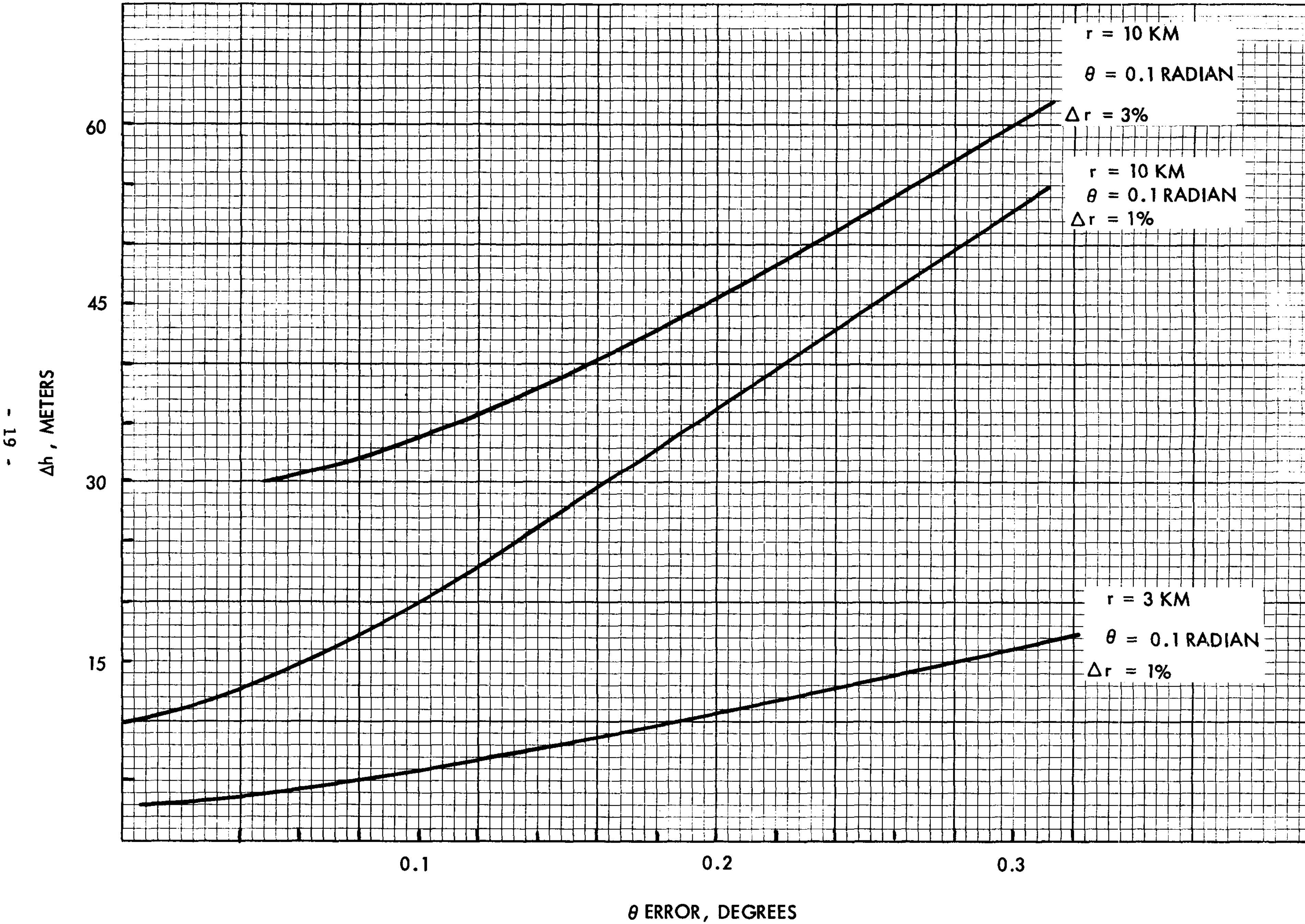


FIGURE 6. ALTITUDE ERROR VERSUS ELEVATION ANGLE ERROR

for this method. A precision limiting factor is an effective θ error caused mainly by the uncertainty in elevation angle to the actual point to which range is being measured.

The curves indicate that at ranges less than 20 km for a 10-20 meter relative altitude error of a single measurement: (a) elevation angle uncertainty with respect to "local vertical" should not exceed 0.1° for an 0.1 radian elevation angle; (b) range accuracy need be no better than 1%. Also, the curves show that larger elevation angles cause larger altitude errors for the same range. However, a series of elevation angle readings taken on the same observable as the Rover passes by it may reduce this error.

It is preferable that neither of the error components in Equation (1) dominate the other. Hence,

$$r\Delta\theta \simeq \theta\Delta r = \theta kr$$

or

$$\frac{\Delta\theta}{k\theta} = 1$$

where k is the fraction error in measuring range. The relation (Figure 7) shows, for example, that there is no need to use better than a 2% ranging device if the effective elevation measurement error exceeds 0.1°.

To obtain an idea of achievable accuracies with this equipment, note that the TV camera accuracy is expected to be between 0.02° and 0.1°. Most inclinometers are accurate to 0.1° over small angles, and the odometer error may exceed 1% under certain circumstances.

Altitude error propagation due to successive altitude measurements at constant range to the observable is discussed in Appendix B.

The landmark navigation method, applied in a vertical plane, is implied in Method No. 1. It is necessary only to move some distance in the general direction of the sighted point, not necessarily to reach it. A true elevation

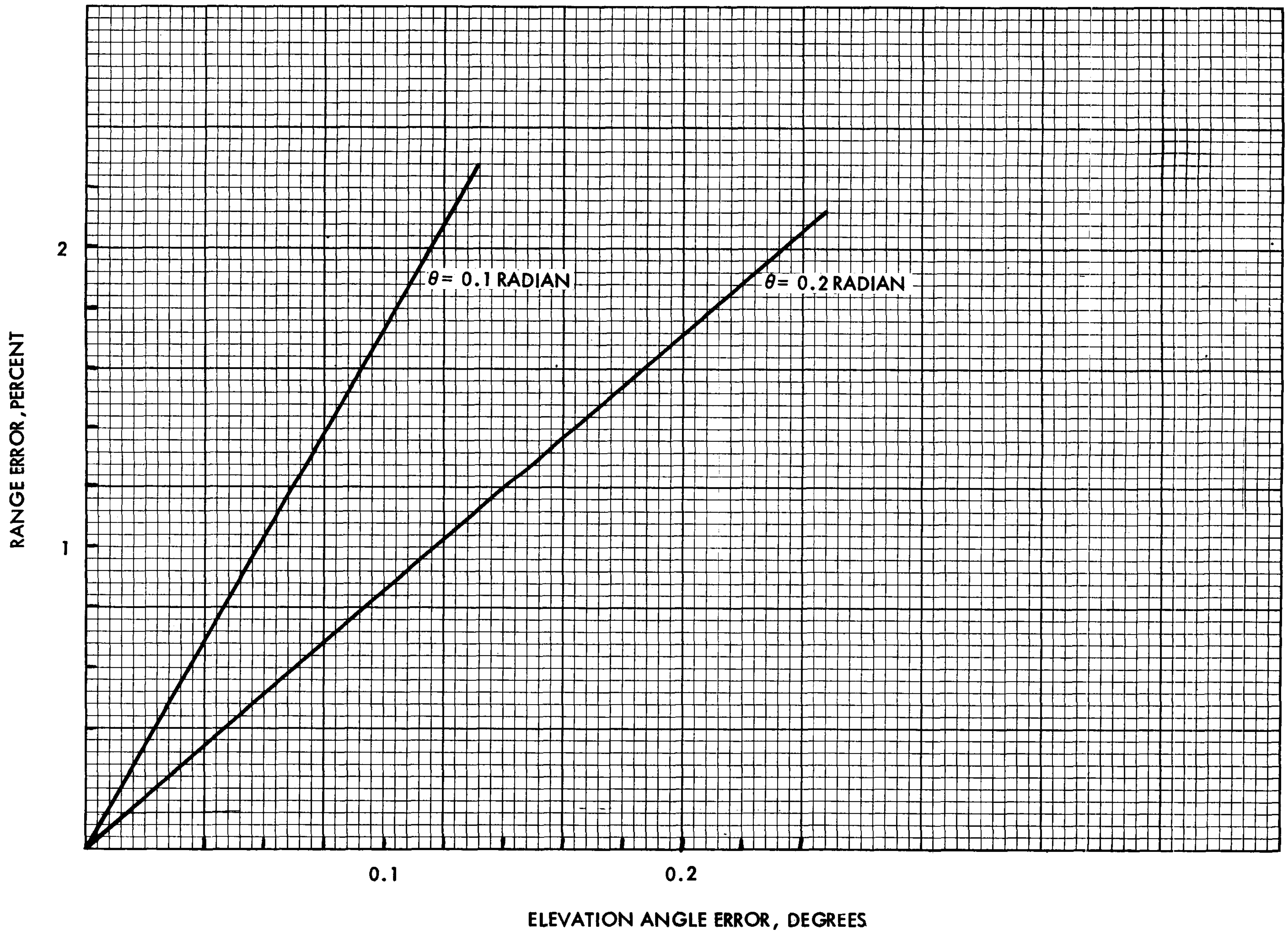


FIGURE 7. ERROR IN MEASURING RANGE VERSUS ELEVATION ANGLE ERROR

reading of the observable, taken at each of two vehicle locations, together with the odometer reading, determines vehicle change in altitude between the two positions, and altitude of the observable with respect to the two vehicle positions. Accuracy of this method depends on elevation angle and distance measuring accuracies and, in a complicated manner, on orientation of the sighted point with respect to the vehicle path.

<u>Method No. 2. Inclinometer Upgrading & Addition of Special Range Finder</u>

Figure 8 is Figure 6 on an expanded scale. The curves show (a) a 10-meter relative altitude error requirement can be met for ranges up to 30 Km and elevation angles up to 0.2 radians with a ranging accuracy of 0.1% and elevation angle accurate to 1 min., (b) little improvement in altitude accuracy results from a range accuracy better than 0.1%, (c) an angular elevation increase from 0.1 to 0.2 radians increases altitude uncertainty by 15%, (d) a one-meter relative altitude accuracy imposes, for example, the stringent combination of requirements:

$$r < 15 \text{ Km}$$

$$\theta \leq 0.1 \text{ Rad.}$$

$$\Delta\theta \leq 10 \text{ Sec.}$$

$$\Delta r \leq 0.05\%$$

(e) the instrumentation of Method No. 1 must be improved to obtain this precision.

Addition of a special rangefinder such as a laser or perhaps an RF doppler ranging device is necessary to meet a 1-10 meter accuracy requirement. Also, a precision 2-axis inclinometer or level must be included in the instrument package. This is a basic instrumentation requirement for precision altitude determination.

Laser development has progressed sufficiently to seriously consider its use on an advanced Lunar Rover. Consider, for example, a laser with the

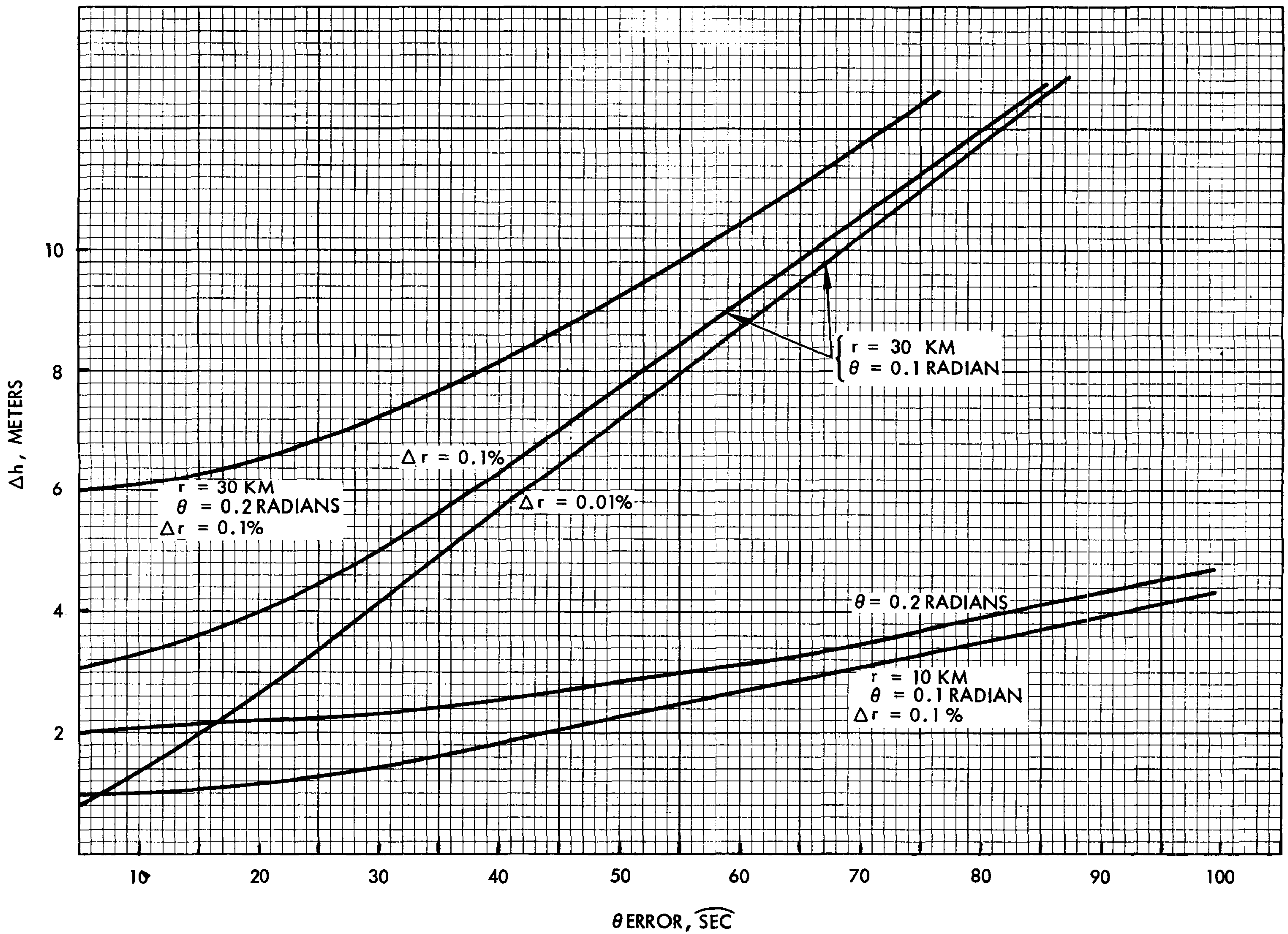


FIGURE 8. ALTITUDE ERROR VERSUS ELEVATION ANGLE ERROR

characteristics summarized in Table 1, Appendix A. A comparable laser is under development by RCA for use on a Lunar Orbiter. It is planned to be operational in less than two years. For landmark ranging purposes up to a range of 30 km, the weight of the laser could be reduced to less than 30 lbs. and the average power to less than 10 watts. Actually, power is needed only during the few seconds taken to make an actual range measurement. Neither standby power nor warmup period is needed.

Time has not permitted investigation of the characteristics of inclinometers in the 20-60 $\widehat{\text{sec}}$. accuracy region. In their Lunar Rover vehicle design study, Bendix mentions a 2-axis inclinometer accurate to 1 $\widehat{\text{min}}$. Very accurate inclinometers can be constructed as evidenced by the < 0.001 $\widehat{\text{sec}}$. instrument built by Hughes Research Labs.

ALTERNATIVE TECHNIQUES FOR METHODS NO. 1 AND NO. 2

These two relative altitude measuring methods use the same principles of (a) establishing local vertical, (b) measuring elevation angle of an observable with respect to the local horizontal, and (c) measuring a distance (either to the observable or that travelled by the Rover). Techniques other than those already considered are available for performing this procedure and will now be discussed.

Inertial Platform

An inertial platform may be substituted for the precision inclinometer and distance measuring device. A star tracker (or sun sensor), three precision accelerometers and three gyros would be used with a three-gimbal platform to provide initial alignment to determine true North (i.e. the direction parallel with the moon's spin axis), and to maintain local horizontal.

Advantages of the platform are: (1) The accelerometers, if of the double integrating type, provide distance travelled and change in altitude.

The latter measurement is, however, not accurate enough for our purposes. (2) A very accurate local vertical is maintained for several hours of travel. (3) A precision reference direction is held for many hours. (4) Accuracy of stable platforms in maintaining local vertical and direction is adequate for any foreseeable altitude measuring requirements. For example, 0.1° per hour gyros on a closed loop stable platform are quite adequate for purposes of initial alignment and maintenance of local horizontal and heading reference. Also, the longitudinal and lateral accelerometers would replace the odometer in addition to measuring distance travelled in a direction perpendicular to the reference heading. A $10^{-6}g_{(earth)}$ accelerometer can result in a maximum error rate of 60 mtrs/hr. Neither the 0.1 deg/hr gyro nor the $10^{-6}g$ accelerometer would be classed as really precision instruments. (Hughes is developing a nano-g accelerometer.)

Disadvantages of an inertial platform are: (1) Relative altitude probably cannot be obtained with sufficient accuracy from the vertical accelerometer. For example, a precision accelerometer ($10^{-7}g_{(earth)}$ will barely sense an altitude error rate of 7 meters/hour. Therefore, a landmark sighting instrument will be necessary. To be consistent with the accuracy of the platform in establishing local horizontal, the sighting instrument arrangement must be very accurate in sensing the elevation angle of the observable, e.g., perhaps a laser. But if a laser or other instrument of equivalent accuracy is needed with the platform, the platform might as well be replaced with an inclinometer for this purpose. (2) Use of a device as complex as an inertial platform in these conditions is rather frightening. Operational problems such as temperature control, checkout, remote alignment, time for alignment, reliability for one year of operation, etc., would be very serious. (3) Platforms are expensive. (4) Finally, platforms are designed principally for rapidly moving reference systems -- not for use on very slow moving vehicles like oxcarts.

Characteristics and the theory of operation of inertial platforms can be obtained from many vendors such as Minneapolis-Honeywell, Litton, Kearfott and Bendix. The theory is thoroughly developed in Reference 27.

Sun Seeker Plus Star Tracker

Another technique is to use the equivalent of a theodolite (or sun sensor). If latitude and longitude are known with reasonable accuracy, local lunar time is sufficient, with one star sighting, to establish selenocentric vertical; i.e., with respect to some "geometric" lunar center. The RSS error, ϵ, in local horizontal would be

$$\epsilon = \left(L_1^2 + L_2^2 + \delta^2\right)^{\frac{1}{2}}$$

where L_1 and L_2 are latitude and longitude uncertainties and δ is the sighting error. For example, if the TV camera is used as a theodolite with an error of 0.05° and $L_1 = L_2 = 1$ km

$$\epsilon = \left[\left(\frac{1}{1740}\right)^2 + \left(\frac{1}{1740}\right)^2 + \left(\frac{.05}{57.3}\right)^2\right]^{\frac{1}{2}} = 0.0012 \text{ rad. } (0.068^\circ),$$

where the camera contributes a little more error than the latitude and longitude errors together, under these assumptions. The accuracy of this method depends on navigation accuracy. It should be noted that most relative elevation determination schemes are equally applicable to the local navigation problem.

Star Seekers

A modification of the previous technique is to employ three star seekers. The procedure would be to measure elevation and azimuth angles to three stars. The stars should be oriented to form a triangle, each star separated from the other two by at least 30° to minimize errors. Error in Rover elevation obtained by this method is

$$\Delta h \geq r\Delta\theta$$

where r = lunar radius and $\Delta\theta$ = error in angle measurement.

This method implies the use of a gyro or TV star picture for heading reference to assist in star location.

Alternatively, a star comparator could replace the three star seekers. A star comparator is a device that matches magnitude and relative orientations of the stars in an observed field of stars with a reference field. The device has been built, but current status has not been ascertained.

Foucault Pendulum and Gyrocompassing

Finally there are two related and rather "far out" versions of methods No. 1 and No. 2. They are gyrocompassing and the Foucault pendulum, or Foucault gyro. Gyrocompassing may be considered as the damping of a Foucault gyro. A Foucault gyro is essentially a horizontal pendulum. Rover elevation could theoretically be measured as suggested by Savet (Ref. 7) either by measuring the period of the undamped pendulum or the displacement of the damped pendulum. Both depend on local gravity. The following calculation indicates the difficulties that would be encountered with these methods.

The period of an undamped gyro in a gyrocompassing mode is given by

$$T = 2\pi\sqrt{\frac{H}{Mg_m \ell\, \omega_o \cos\lambda}}\,, \qquad (2)$$

where H = angular momentum of gyro

M = gyro mass

ℓ = pendulous arm

ω_o = moon rate of rotation,

λ = lunar latitude

g_m = lunar gravity.

For example, using an HIG5 gyro, assuming a rotor mass of 1 kg. and pendulous arm of 10 cm, the period on the moon is

$$T = 2\pi \sqrt{\frac{10^5 \text{gm} \ \frac{\text{cm}^2}{\text{sec}}}{10^3 \text{gm} \times 167 \ \frac{\text{cm}}{\text{sec}^2} \times 10\text{cm} \times 2.66 \times 10^{-6} \ \frac{\text{rad}}{\text{sec}}}}$$

$T = 960$ seconds.

From Eq. (2), $\Delta t = \frac{t}{2} \ \frac{\Delta g}{g_o}$.

Gravity variation with elevation is

$$\Delta g = 2 \ g_o \ \frac{\Delta r}{r_o}$$

$$\Delta t = t \ \frac{\Delta r}{r_o}$$

For an elevation change of 10m, $\Delta t = 0.005$ seconds.

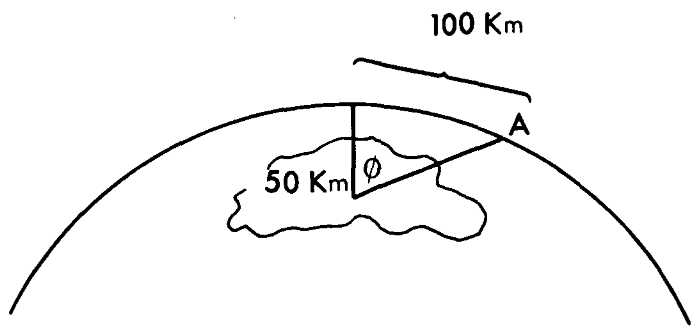
It does not seem feasible to remotely measure 10^3 sec. to that accuracy. If further study were to show feasibility of such measurements, Savet's analysis should be carefully checked. This is necessary because two errors occur elsewhere in the same report. One is a factor of 57.3 (Eq. 10b), and the other is a factor of 840 (Eq. 31).

A new type of north-seeking gyro with a potential accuracy of 1 sec. is being developed at Hughes Research Laboratories. It is not far enough along in the development stage to be considered here.

A special correction to the local vertical may be necessary for the effect of mascons in the case of precision altitude measurement, as in Method No. 2. A sample computation to indicate the expected magnitude is presented in the next section.

Deviation of the Local Vertical

According to Dr. R. L. Forward of Hughes Research Laboratories, a reasonable mascon-caused gravitational anomaly of 20 eötvos units in gravity gradient may be expected at 50 Km altitude. As a first approximation, the resulting change in lunar gravity at the surface is 0.003 ft/sec^2. An estimate of this effect on the local horizontal can be obtained as follows. Assume the mascon is lens shaped, 200 km long with center 50 km below the surface. Let the increase in gravity at a point directly over the center of the mascon be .003 ft/sec^2 = 0.1 cm/sec^2. The effect at a point above the edge of the mascon (point A) is



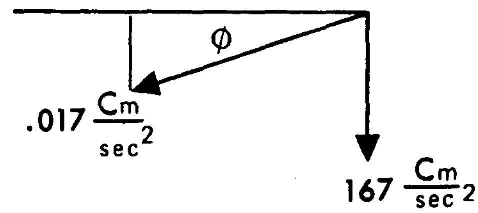
$$\Delta g_A = 0.1 \times \left(\frac{50}{120}\right)^2 = .017 \text{ cm/sec}^2$$

which is directed downward from the horizontal at an angle

$$\varphi = \sin^{-1}\left(\frac{50}{120}\right) = 25^{\circ}$$

The horizontal component is

$$\Delta g_h = .017 \cos \varphi = .0155$$



Deviation of local horizontal is

$$\frac{.0155}{162} \times 3600 \times 57.3 = 20 \widehat{\sec}.$$

For most altitude measurements, this is not serious. Knowledge of the existence of the mascon would permit correction of the local vertical, if desired.

Method No. 3. Use of a Lunar Orbiter

A lunar orbiter can be used for, among other things, both relative and absolute elevation determination. Consider first its application to determination of the relative altitude between Rover positions. This can be accomplished either by doppler radar tracking of the orbiter from the Rover, or possibly by laser altimeter measurement of Rover altitude from the orbiter. Only the radar tracker will be described here. See the section on absolute altitude determination. It is shown there that altitude difference between the orbiter and Rover can be measured to an accuracy of better than 10 meters.

An additional source of error, non-instrumental, arises in this method. It is due to mascon-caused irregularities in an otherwise known orbit. The following rough calculation indicates that a fairly large mascon would cause a relative altitude error of only .04 meter/km, on the lunar surface.

It was estimated above that a mascon can cause a change in lunar gravity of 0.003 ft/sec^2 at the surface, or 0.001 ft/sec^2 at orbital distance from the mascon. The effect on the maximum rate of departure of the orbiter from its originally "circular" orbit is of concern here. Although obtaining this rate requires solution of a version of the 3-body problem, the following approach may give an order of magnitude solution, which is sufficient for the present purpose.

The acceleration, a, acting on the orbiter is

$$a = \frac{\mu}{r^2} \tag{3}$$

$$v^2 = \frac{\mu}{r} \tag{4}$$

$$2v\Delta v = \frac{r\Delta\mu - \mu\Delta r}{r^2} = \frac{\Delta\mu}{r} - \frac{\mu\Delta r}{r^2} \tag{5}$$

From Eq. (3), $\Delta\mu = \mu \frac{\Delta a}{a}$ for constant r. Δv due to $\Delta\mu$ is for v = 5350 ft/sec. and holding r constant, $\frac{\Delta v}{\Delta\mu} = \frac{1}{2}v \frac{\Delta a}{a} = 0.5$ ft/sec.

Now if we decrease r to a value such that the greater centrifugal acceleration balances the larger gravity acceleration, we have

$$v^2 = ar$$

$$2v\Delta v = a\Delta r + r\Delta a$$

$$\Delta r = \frac{2 \times 5350 \times 0.5 - 5.85 \times 10^6 \times .001}{5.3} \simeq -100 \text{ ft.}$$

To obtain the error in Rover relative altitude measurement due to mascon-caused orbit altitude variation, we need the distance travelled by the orbiter to reach its maximum change in altitude. A rough estimate of this can be obtained by assuming the extra acceleration caused by the mascon is a constant over a sufficient distance along the orbit to effect the change in altitude.

$$\Delta r = \frac{1}{2} t^2 \Delta a$$

$$t = \left(\frac{2 \times 100}{.001}\right)^{\frac{1}{2}} = 450 \text{ sec.}$$

Slope of the orbit is $\frac{\Delta r}{\text{distance travelled}} \simeq - \frac{100}{450 \times 5400} = 0.04 \frac{\text{meters}}{\text{km}}$

This is small, <u>if</u> all the assumptions are reasonably valid. The error would become negligible if knowledge of the shape of the orbit could be improved upon as a result of tracking data from earth.

SELENODETIC (ABSOLUTE) ELEVATION

An orbiter would seem to be a logical way to determine absolute or selenodetic elevation because absolute elevation is referenced to the center of mass and the orbiter rotates about the center of mass. Altitude of the orbiter with respect to the Rover can be obtained by a method described below. The difficult problem is to obtain the radius of the orbit and to correct for orbital anomalies.

RF Tracking of Orbiter from Rover

The method described here tracks the orbiter from the location where the absolute elevation is desired. Tracking can be accomplished either with a doppler radar or with a laser. Of the two methods, the radar is easier to implement because a wide-beam antenna can be used, obviating the need to servo the antenna. Laser beams are so narrow that directional control of the beam would be necessary in this application.

A version of the doppler method is briefly outlined below, with calculations. In this method, a constant frequency is transmitted from the Rover via a two-element array (e.g., double turnstile). The orbiter receives the signal as a variable frequency because of the doppler shift due to orbiter velocity change with respect to the transmitter.

Let v = satellite velocity

h = satellite altitude above the Rover

f_o = transmitted frequency

f = received frequency

c = velocity of light.

For simplicity, it is assumed the satellite passes directly over the Rover, but the same principles apply if it does not. Then,

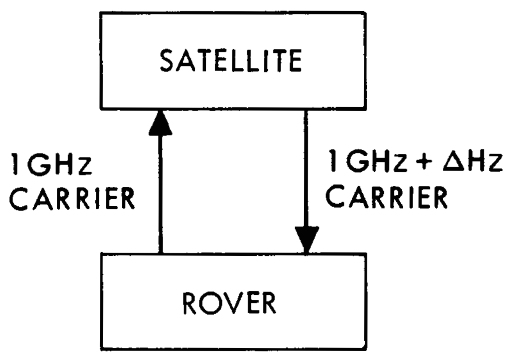
$$f = f_o \left(1 + \frac{v}{c} \cos \theta\right)$$

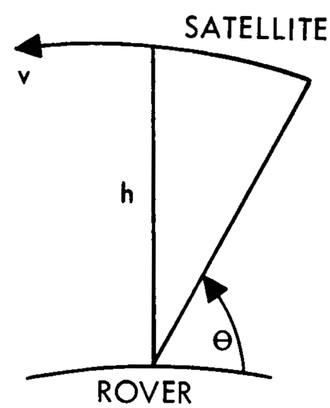
$$\frac{df}{dt} = f_o \frac{v}{c} \sin \theta \, \dot{\theta}$$

$$\dot{\theta} = \frac{v}{h} \text{ when satellite is directly above Rover}$$

$$\frac{df}{dt} = f_o \frac{v}{c} \sin \theta \frac{v}{n} = \frac{f_o v^2}{cn}$$

This is the maximum apparent acceleration of the orbiter.





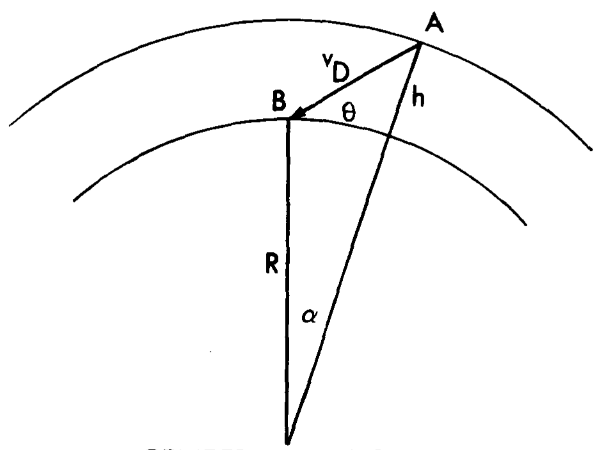
For a 1 Ghz carrier and a 30 Km orbit, the maximum apparent acceleration or rate of change of two-way doppler shift is 460 Hz/sec. See Appendix D for link calculations. In Appendix D it is seen that if two seconds of sampling time at 50 samples per second, at an altitude of 30,000 meters, the 100 data point accuracy is 3150:1 or a range accuracy of 30,000/3160 $\simeq$ 10 meters.

The effect of ground clutter has not been considered. It should be small for this type of antenna because of the low elevation angle of any local hills with respect to the Rover, and because of the poor reflectivity of the dry soil.

Sampling time probably could be made longer, thus improving accuracy. However, other factors such as oscillator phase jitter, degradation due to moon-to-earth transmission etc. have been ignored.

The above assumes orbiter velocity is known; however, it is not. There remains an orbit determination problem.

Additional information resides in the doppler data when that data are obtained throughout a large range of the elevation angle φ, say $\pm 45^o$. Thousands of samples can be obtained during a single pass from which a very smooth curve of v_D vs time can be drawn. One way to use this data to assist in the orbit determination problem is now outlined.



Referring to the diagram, the doppler velocity v_D is

1) $$v_D = v \cos \alpha \cos \theta$$

2) $$\frac{dv_D}{dt} = -v\,(\sin \alpha \cos \theta\, \dot{\alpha} + \cos \alpha \sin \theta\, \dot{\theta})$$

3) $$\frac{\dot{v}_D}{v_D} = -(\dot{\alpha} \tan \alpha + \dot{\theta} \tan \theta)$$

The small angle α is known in terms of R, h and θ from the triangle ABC, i.e., from law of sines.

4) $$\sin \gamma = \frac{R}{R + h} \cos \theta$$
$$\gamma = \frac{\pi}{2} - (\alpha + \theta)$$

v_D is a measured function of time

$\dot{v}_D$ is known from the slope of the smooth $v_D = f_1(t)$ curved

$\dot{\alpha}$ is known from the orbit period.

Then

5) $\frac{\dot{v}_D}{v_D} = f_2(t)$ is obtained.

Next,

6) $\theta = f_3\left(\frac{\dot{v}_D}{v_D}\right)$ is found from (3) and (4).

From (5) and (6) the relation

7) $\theta = f_4(t)$ can be computed.

From (7) and $v_D = f(t)$, we have

8) $v_D = f(\theta)$.

Then from (1) and (8) an average or smoothes value of v is available. Finally, knowing orbit period P very accurately, the radius of the orbit is obtained,

$$R_o = \frac{Pv}{2\pi}$$

The selenodetic elevation is $R_o - h$.

Corrections to R_o must be made for gravitational anomalies and triaxiality of the moon. An error analysis of the above procedure is necessary. A velocity accuracy in the neighborhood of 0.02 ft/sec is required to reduce the absolute altitude error to 10 meters.

Presumably additional information could be obtained by tracking the orbiter from earth. For example, Rover position and altitude could be derived by differencing range and velocity between earth-orbiter, earth-Rover and Rover-orbiter. In this manner, small local velocity changes in the vicinity of the Rover would be detected.

Other ways to obtain either orbital velocity or radius were briefly investigated but none were satisfactory. For example, the value of gravity at the orbit radius can be related to velocity, radius and the lunar gravitational parameter. Then by measuring gravity gradient on the orbiter, the radius can be obtained. But the accuracy of 1:20,000 with which gravity gradient can be measured in the orbiter leaves an uncertainty in orbit radius of 70 meters.

Another unsuccessful attempt was to time the travel of the orbiter through some angle as observed from the Rover or the Landing Module. To achieve the required accuracy in velocity, tracking of the orbiter must be accomplished with a precision of 14 $\widehat{\text{sec}}$. through a 90° angle. The 90° traverse must be timed to within an error of $\pm 4 \times 10^{-5}$ parts or 0.14 milliseconds. The accuracies are derived as follows:

v of orbiter $\cong$ 1650 m/sec.

distance travelled through 90° is d $\simeq$ 60,000 meters

average velocity error permitted is $\Delta v = 0.02$ ft/sec.

Then distance error allowed is

$$\Delta d = t\,\Delta v = \frac{60,000}{1650} \times \frac{.02}{3.05} = .24 \text{ meters}$$

Allowable time error is

$$\Delta t = \frac{\Delta d}{v} = \frac{0.24}{1650} = 1.45 \times 10^{-4} \text{ sec. or one part in } 4 \times 10^{-5}.$$

The allowable angular error is

$$\Delta\theta = \frac{\Delta d}{r_{avg}} \approx \frac{.24}{36,000} = 6.7 \times 10^{-5} \text{ rad}$$

$$= 14\ \widehat{\text{sec}}.$$

Other Methods

Three other methods for establishing absolute elevation are:

1. RF and doppler tracking of an antenna on the moon. A version of this is laser tracking of corner cube reflectors placed on the moon, perhaps by the Rover. This subject can more appropriately be treated by JPL because of their experience in long-range tracking, data smoothing and data interpretation. The authors of References 15-18 should be contacted for information on precision, tracking time, etc.

2. Differential Ranging (Ref. 21). The Abstract of this Reference is as follows:

> "A method of navigating at lunar distances is developed which depends on differential ranging to two lunar stations from a triplet of MSFN stations on earth. The method can easily be applied to the case of navigating a lunar roving vehicle or flying unit relative to its parent spacecraft. In conjunction with on-board CSM navigation using landmarks, the method could also be used to navigate an LM to high gate allowing the major errors to be removed while the engine is operating efficiently."

3. Use of a very long base interferometer (VLBI). If an angular accuracy of 0.005 $\widehat{\text{sec}}$. is assumed for this device, the accuracy at lunar distance would be ± 11 meters. At the suggestion of JPL, this subject was not pursued.

In Table II it is assumed: (a) the Rover has a normal or basic complement of instruments consisting of a TV camera, an odometer, an inclinometer and some kind of heading reference; (b) position and heading are known; and (c) when the TV camera is used, it is referenced to the inclinometer.

Small relative numbers mean the item is most favorable; e.g., 1 means precision accuracy or least cost. In the accuracy column, 4 means perhaps 50 meters, 1 means 5 meters.

Performance sensitivity is a measure of probable degradation resulting from environmental and operational effects. It is related to reliability.

Additional auxiliary hardware refers to the necessity for extra hardware in the lunar environment such as an orbiter or special ranging site.

Three star sensors imply need for a gyro or TV star picture for heading reference to assist in star location.

ALTITUDE MEASURING INSTRUMENTS	TV INCLI-NOMETER ODOMETER	LASER PRECISION INCLINOMETER	STABLE PLATFORM	3 STAR SENSORS OR STAR COMPARATOR	STAR SENSOR SUN SEEKER LAT & LONG.	GYRO COM-PASSING	FOUCAULT PENDULUM	R.F. TRACK-ING FROM EARTH	DIFF. RANG-ING	VLBI	ORBITER (a) RF TKG. (b) LASER TKG
RELATIVE ACCURACY	2	0	3	3	3			1		0	(a) 0 (b) 0
RELATIVE COMPLEXITY	0	1	4	4	2			0	2	?	1 2
TIME FOR FIX	0	0	2	2-3	1	4	4	4+	4+	0	1 & 0
STATE OF THE ART	YES	YES	YES	YES	YES	NO	NO	YES	NO	NO	YES YES
PERFORMANCE SENSITIVITY	0	1	3	3	2	POOR	POOR	0	1	0	0 1
RELATIVE ELEV. AVAIL.	YES	YES	YES	YES	YES			YES	YES	YES	YES
ABSOLUTE ELEV. AVAIL.	NO	NO	NO	YES	YES			YES	YES	YES	YES
USEFUL FOR HOR. NAV.	YES	YES	YES	YES	YES			AS BACKUP?		YES	YES
RELATIVE COST	0	3	2	2	1			1	2	?	4 (UNLESS 4+USED FOR OTHER PURPOSES)
ADDITIONAL VEH. OR AUX. HARDWARE REQUIRED	NO	YES	YES	YES	YES	YES	YES	NO	YES	NO	YES

Small relative numbers means the item is most favorable such as least cost, precision accuracy, etc.

TABLE II. Preliminary Evaluation of Elevation Determination Methods

	ADVANTAGES	DISADVANTAGES
TV Camera Inclinometer Odometer	1) Simple, Reliable, operationally easy 2) No extra cost or extra onboard equipment 3) Can use for horizontal navigation 4) Info immediately available	1) Poor accuracy
TV Camera Laser Precision Inclinometer	1) Very accurate, reliable, operationally easy 2) Info immediately available 3) Can use for horizontal navigation	1) Costly
Stable Platform	1) Complete navigation system, even in case of obstacle avoidance maneuvers 2) Can replace vehicle odometer heading ref. instrument and inclinometer	1) Complex, unreliable for one year operation in Lunar environment 2) Poor accuracy for elevation measurement unless frequent time consuming fixes
3 Star Seekers or Star Field Comparator	1) A basic, very accurate horizontal navigation system	1) Time to locate 3 stars or to match star fields 2) Fairly complicated, delicate instruments 3) Poor elevation determination accuracy
Star Seeker Sun Sensor Longitude Latitude	1) A very accurate horizontal navigation method 2) Fairly easy operationally	1) Not applicable for sun altitudes exceeding 70^{o} 2) Poor elevation determination accuracy
Gyrocompass Foucault Pendulum	None with presently existing equipment	Not "state of the art" to yield acceptable accuracies
RF Tracking	1) Very accurate for 3 dimensional fix 2) No extra onboard equipment 3) Small extra cost 4) Absolute altitude available	1) Tracking time too long for relative elevation determination or for horizontal navigation
Differential Ranging	1) Probably could be as accurate as the RF tracking method	1) Tracking time too long 2) Extra ranging station on moon
VLBI	1) No extra onboard equipment	1) Does it exist?
Orbiter (a) RF Tracking	1) Absolute altitude available 2) Very accurate position & elevation from determination	1) Cost 2) Extra equipment (the orbiter)
(b) Laser Tracking	1) Orbiters can also be used for mapping, gravity gradiometry, photos	1) Cost 2) Laser tracking problem 3) Extra equipment (the orbiter)

TABLE III. COMPARISON OF ELEVATION DETERMINATION METHODS

GENERAL CONCLUSIONS

The following partially subjective comments result from the study:

1. The basic Rover instruments probably can be made adequate for most of the shorter trips. Cost and reliability are the attractive features here.

2. A TV camera with a ranging (and picture marking) laser with an 0.1° or better inclinometer is quite sufficient instrumentation for precision relative elevation determination. The matter of laser cost is the important factor, however.

3. Gyrocompassing of any form should not be given further consideration unless a new type of instrument becomes available.

4. Stable platforms and vertical or directional gyros should be avoided if possible because of reliability and thermal control problems. Platforms also present operational problems.

5. Orientation of landmarks in landmark navigation should be chosen to lie within $\pm$ 60° of the Rover path. Preferably they should be at an initial range exceeding the step length.

6. Tables 1 and 2 should be of assistance in choosing between the various instrumentation arrangements.

RECOMMENDATIONS

The following are some of the areas needing further study.

1. Before comparisons can be made between navigation and guidance schemes, block diagrams of two or three of the most promising should be drawn. These should be in sufficient detail to count components such as amplifiers and multipliers, and to determine degree of redundancy, instrument specifications, etc. The systems should be compared from points of view such as reliability, availability of instruments, operational problems, data rates and procedure for initial alignment.

2. Certain particular instrumentation problems need to be considered in the next phase. Some of these are:

a) Availability of inclinometers in the 20-60 $\widehat{\text{sec}}$. range, with proper behavior on a rough ride.

b) Accuracy and feasibility of using various types of odometers, particularly the microwave doppler type.

c) How to mount the TV camera, i.e., make it pendulous? It may need three servos.

d) What kind of a sun seeker arrangement should be used to provide at least a $30^{\circ} \times 30^{\circ}$ f.o.v.?

3. Look into the low latitude attitude reference problem where neither sun nor earth sensors can be used. Are operations necessary at noon time? Is the degradation caused by use of TV star pictures in combination with a directional gyro acceptable?

4. Look into particular and multiple uses for lasers.

a) A design layout and optical analysis should be made of the laser-inclinometer-TV camera arrangement on the Rover that was suggested in

Appendix A. The purpose would be to establish feasibility of that suggestion.

b) Determine feasibility of laser uses for such functions as night driving, obstacle detection, etc.

5. Finish the landmark navigation study initiated at Hughes. This is basically an error analysis involving extension of the study to include:

a) Several landmarks and several observations of each in relation to their distribution with respect to the vehicle track.

b) The effect of course deviations.

c) Landmark distribution probabilities.

6. The following miscellaneous navigation and guidance problems should be investigated:

a) Navigation through and in a crater.

b) Navigation in the presence of many large rocks.

c) Navigation in very broken and hilly areas.

7. The matter of thermal control should be given consideration in selection of LRV navigation systems. This could be a decisive factor in choice of certain instruments. For example, a 90^{o} turn of the Rover can move a piece of equipment from shade to direct sunlight. Also, most types of gyros would need some heat during the lunar night, which would cause battery drain.

APPENDIX A

LASERS

Potential uses of lasers for lunar explorations are sufficiently numerous to warrent separate comment. Suggested laser applications are: (1) in an orbiter to obtain instantaneous altitude for purposes of mapping the lunar surface (2) in an orbiter to map gravity gradient (3) on a Rover to range to a reflector on an orbiter to obtain Rover position and altitude information and as a direct navigation aid (the orbiter would be there for other purposes in this case) (4) on the Rover for accurate landmark navigation and altitude determination (5) on the Rover for terrain mapping with a scanning laser (6) on the Rover for other possible miscellaneous purposes such as obstacle detection, night driving intermittent illumination, vaporization of lunar material for spectral analysis, and holographic applications.

The following description of a laser for lunar use is from a recent Hughes proposal to NASA (Reference 20).

> "The laser altimeter will be used to measure the distance from an Apollo spacecraft in lunar orbit to the moon's surface. The altimeter will be primarily in support of a metric camera experiment in which it will measure the range to an identified point on the metric camera scanning laser photograph. The altimeter data will also be used to determine broad variations in the moon's topography.
>
> "The laser altimeter will determine range between the Service Module Instrument Carrier (SMIC), where the altimeter will be located, and the lunar surface by measuring the time required for a pulse of light energy to travel to the lunar surface and return. An electro-optically Q-switched Nd:YAG laser

will be used to generate a 100 millijoule, 10 nanosecond pulse of 1.06 micron radiation. A solid state detector of the germanium avalanche type will be used to detect the return pulse.

"The altimeter range measurement will be synchronized to operate in conjunction with the Metric Camera shutter. A retro-reflective prism assembly will be used to send altimeter boresight information to the Metric Camera. This boresight information will indicate the altimeter optical axis to within 100 microradians (20 $\widehat{\text{sec}}$.).

"The altimeter operation will be controlled by electrical signals from the Apollo command module. In addition to triggering in synchronization with the Metric Camera shutter, an automatic mode is available in which the altimeter ranging sequence is triggered automatically every 10 seconds.

"A mechanical shutter activated by a separate photodiode circuit in the altimeter protects the sensitive germanium avalanche detector from direct radiation from the sun."

The specifications of Table A-1 are typical of the present state of the art. The weight and power requirements are rather high for a lunar rover, but a rover does not need a 150 Km range with an accuracy of $1{:}10^5$. If range were reduced by a factor of 5 or 10 and accuracy resolution by an order of magnitude, weight and power could be reduced by a factor of 2 or more.

Figure A-1 is a block diagram of the lunar laser altimeter (Reference 20).

For precision navigation and altitude determination, the following arrangement should be considered. Mount the TV camera, laser, and inclinometer integrally at the base of the mast. This method of mounting improves maintenance of alignment between the camera and laser optical axes in addition to alignment between the laser optical axis and the inclinometer reference. A gimballed mirror high up on the mast serves to reflect the scene into the camera. The precise place to which range is measured by a laser pulse can simultaneously be marked on the TV picture by a method such as that employed in the above described laser altimeter to mark the metric camera photo.

Comparative calculations of range, power, and receiver diameter can be made from Equation (A-1).

$$E_r = \frac{k\, E_T\, \rho D^2}{R^2} \tag{A-1}$$

where

E_r = energy received

E_T = energy transmitted ($\sim$ power)

ρ = reflectivity

D = receiver optics diameter

R = range.

TABLE A-1. SUMMARY OF LASER LUNAR ORBITER ALTIMETER SPECIFICATIONS

PERFORMANCE	
Laser wavelength	1.06 microns
Pulse energy	100 millijoules nominal output
Pulse duration	10 nanoseconds nominal
Pulse repetition rate, maximum	Once very 8 seconds
Pulse synchronization (CAMERA mode)	Within 3 milliseconds of metric camera shutter signal
Transmitter beam divergence	80 percent of energy within 0.2 milliradian cone
Receiver field of view	0.2 milliradian cone
Range resolution	2 meters
Range accuracy	90 percent of rangings within 2 meters
Operating range against lunar surface (for signal probability of 0.99, false alarm rate of 0.01)	40 to 80 nautical miles
Required power to operate	30 watts design goal, 50 watts maximum, at 27.5 volts DC
Flashlamp life	250,000 flashes for 10 percent decrease in laser output nominal
PHYSICAL	
Total weight *	50 pounds design goal, 75 pounds maximum
Dimensions	Wedge shaped package, 16 in. x 24 in. on output face 10 in. x 24 in. on rear face 12 in. maximum depth
ENVIRONMENTAL	
Temperature, operating	-40°F to +180°F
Temperature, storage	-50°F to +200°F
Shock	Per specifications
Vibration	Per specifications

* See Text

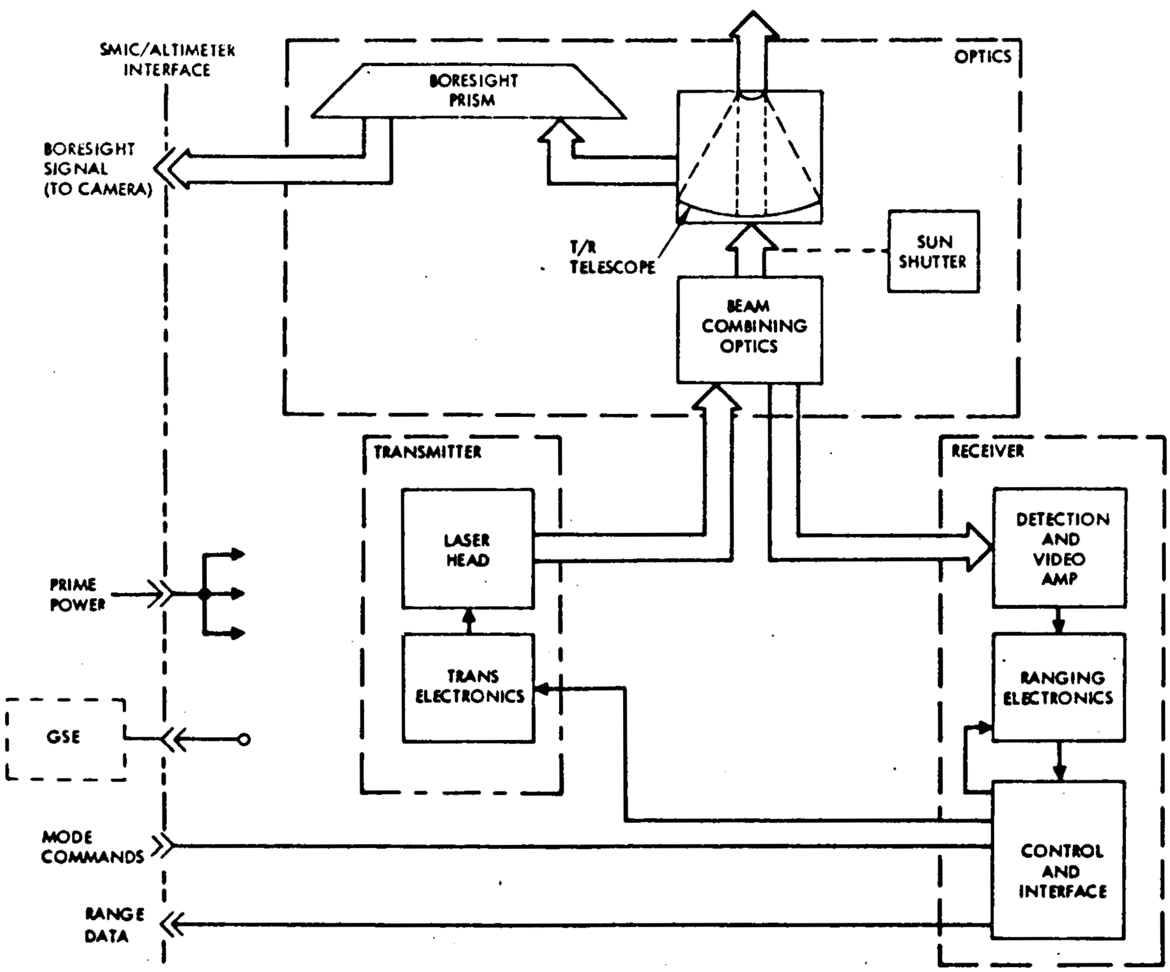


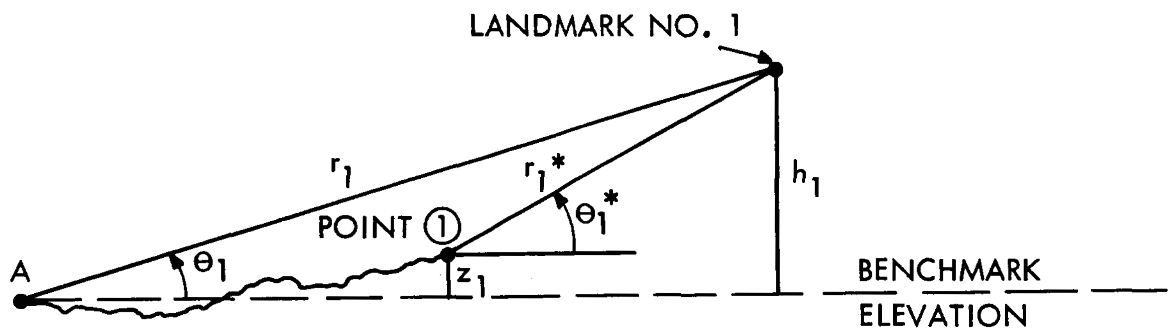
Figure A-1. Simplified Block Diagram of Laser Altimeter

APPENDIX B

ERROR PROPAGATION

Each elevation measurement will have an error. The mean value of the errors will be zero unless a bias occurs in one of the measuring instruments. However, the elevation uncertainty will increase with each measurement, as shown below.

Assume an idealized situation wherein at each stop an observable is available at the same range as the one used for elevation measurement at the previous stop. Assume two measurements are made, one at each of two ranges, of each observable, as shown in the diagram. Assume that range can be measured exactly



and that each angular measurement θ_i has an error η which is Gaussian with mean zero and variange σ_θ^2.

Let r_i be the range from the i-1 point to the i^{th} landmark, and let r_i^* be the range from the i^{th} point to the i^{th} landmark. Let the initial point, A, have elevation z = 0. Then the altitude of the first landmark, h_1, is

$$\frac{h_1}{r_1} = \sin(\theta_1 + \eta) \approx \sin \theta_1$$

Differentiating,

$$\frac{\Delta h_1}{r_1} = \eta \cos \theta_1$$

$$\Delta h_1 = r_1 \eta \cos \theta_1 = \sqrt{r_1^2 - h_1^2}\ \eta$$

$$E(\Delta h_1) = 0$$

$$E(\Delta h_1)^2 = \sigma_{h_1}^2 = \left(\sqrt{r_1^2 - h_1^2}\right)^2 \sigma_\theta^2 = (r_1 \cos \theta_1)^2 \sigma_\theta^2$$

At point 1

$$\sin (\theta^* + \eta) = \frac{h_1 - z_1}{r_1^*} \approx \sin \theta_1^*$$

Since h is approximately known from the first measurement,

$$\sin \theta_1^* = \frac{r_1}{r_1^*} \sin \theta - \frac{z_1}{r_1^*}$$

$$z_1 = r_1 \sin \theta_1 - r_1^* \sin \theta_1^*$$

Differentiating to obtain the uncertainties at point 1

$$\eta \cos \theta_1^* = \frac{\Delta h_1 - \Delta z_1}{r_1^*}$$

$$\Delta z_1 = \Delta h_1 - r_1^* \eta \cos \theta_1^*$$

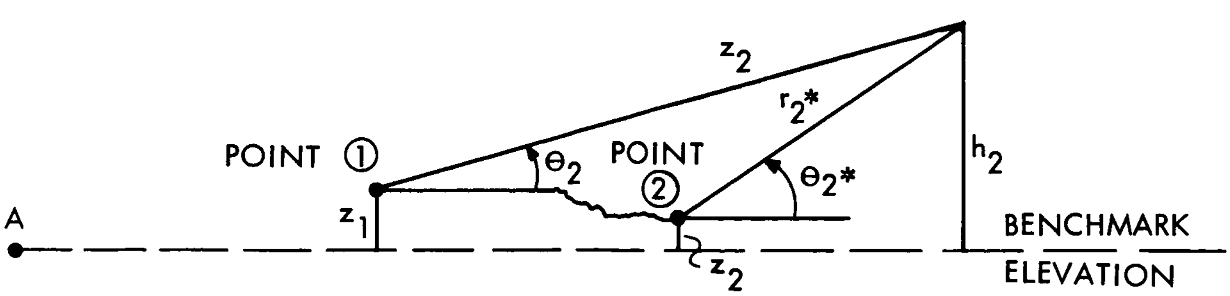
$$E(\Delta z_1) = 0$$

$$E(\Delta z_1^2) = \sigma_{h_1}^2 + (r_1^* \cos \theta_1^*)^2 \sigma_\theta^2 = \left\{(r_1 \cos \theta_1)^2 + (r_1^* \cos \theta_1^*)^2\right\} \sigma_\theta^2$$

z_1 then has the mean value $r_1 \sin \theta_1 - r_1^* \sin \theta_1^*$ and the variance

$$\left[(r_1 \cos \theta_1)^2 + (r_1^* \cos \theta_1^*)^2\right] \sigma_\theta^2$$

Now from point 1 a second landmark is selected at range r_2, elevation angle θ_2, altitude h_2



$$\sin(\theta_2 + \eta) = \frac{h_2 - z_1}{r_2} \approx \sin\theta_2$$

$$h_2 = r_2 \sin\theta_2 + z_1$$

$$E(h_2) = r_2 \sin\theta_2 + r_1 \sin\theta_1 - r_1^* \sin\theta_1^*$$

$$\eta \cos\theta_2 = \frac{\Delta h_2}{r_2} - \frac{\Delta z_1}{r_2}$$

$$\Delta h_2 = r_2\, \eta \cos\theta_2 + \Delta z_1$$

$$E(\Delta h_2) = 0$$

$$E(\Delta h_2)^2 = \left[(r_2 \cos\theta_2)^2 + (r_1 \cos\theta_1)^2 + (r_1^* \cos\theta_1^*)^2\right]\sigma_\theta^2$$

Now move to point 2 and sight again on landmark 2.

$$\sin(\theta_2^* + \eta) = \frac{h_2 - z_2}{r_2^*} \approx \sin\theta_2^*$$

$$z_2 = h_2 - r_2^* \sin\theta_2^*$$

$$z_2 = r_2 \sin\theta_2 + r_1 \sin\theta_1 - r_1^* \sin\theta_1^* - r_2^* \sin\theta_2^*$$

$$\eta \cos\theta_2^* = \frac{\Delta h_2}{r_2^*} - \frac{\Delta z_2}{r_2^*}$$

$$\Delta z_2 = \Delta h_2 - r_2^*\, \eta \cos\theta_2$$

$$E(\Delta z_2) = 0$$

$$E(\Delta z_2)^2 = \sigma_{h_2}^2 + (r_2^* \cos\theta_2)^2\, \sigma_\theta^2$$

$$= \left[(r_1 \cos\theta_1)^2 + (r_2 \cos\theta_2)^2 + (r_1^* \cos\theta_1^*)^2 + (r_2^* \cos\theta_2^*)^2\right]\sigma_\theta^2$$

Similarly, z_n has the mean value

$$E(z_n) = \sum_{i=1}^{n} (r_i \sin\theta_i - r_i^* \sin\theta_i^*)$$

and variance

$$E\left[z_n - E(z_n)\right]^2 = \sum_i \left[(r_i \cos\theta_i)^2 + (r_i^* \cos\theta_i^*)^2\right]\sigma_\theta^2$$

For example, let $\cos\theta = 1$, $r_i^* = \frac{r_i}{2}$ in all measurements and all r_i equal; then

$$\sigma_z = \sigma_h \sqrt{1.5\,\eta}$$

Let the error in each measurement be 3 meters on a range of 3 km. The distance that can be travelled before the uncertainty (variance) in altitude as referenced to the starting (benchmark) altitude reaches 20 meters is 3n kilometers where n is given by

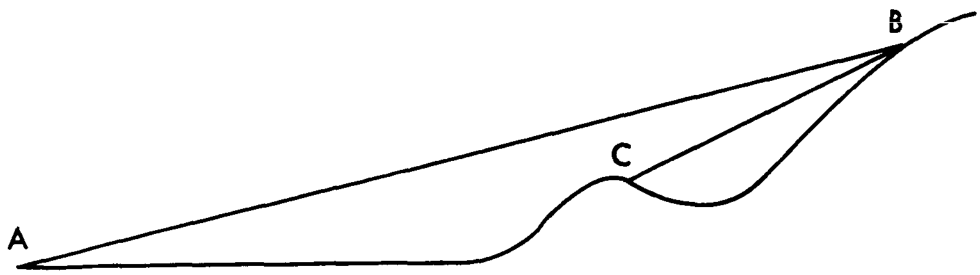
$$3\sqrt{1.5\,n} = 20$$

$n = 30$ and distance travelled is

$d = 30 \times 3 = 90$ Km.

The graphs of Figures II-1 to 3 show error growth versus the number of vehicle stops. The calculations are based on the assumption that two elevation angle readings on each landmark are taken at different ranges from it, the distance to each new landmark remaining constant. Distance between stops also is constant for any one curve and is measured to an accuracy of 0.1%. The major contributor to error accumulation for all the curves occurs in the first term of Eq. (1) P.2. It is seen that useful altitude measurements require an elevation accuracy of better than one-half degree.

The general case of n successive sightings on n landmarks at miscellaneous ranges is a statistical problem beyond the scope of the present effort. Comment can be made on successive sightings on a single landmark. It is that elevation accuracy with respect to a benchmark will not improve with repeated measurements taken at decreasing distances. This can be seen from the diagram below.



A sighting on landmark B from reference altitude A will have an error Δh. The error in an altitude measurement of B with respect to C, taken at C, will be less than Δh because the range is less. But the altitude uncertainty of C with respect to A will be greater than Δh because of the error in making the measurement at C.

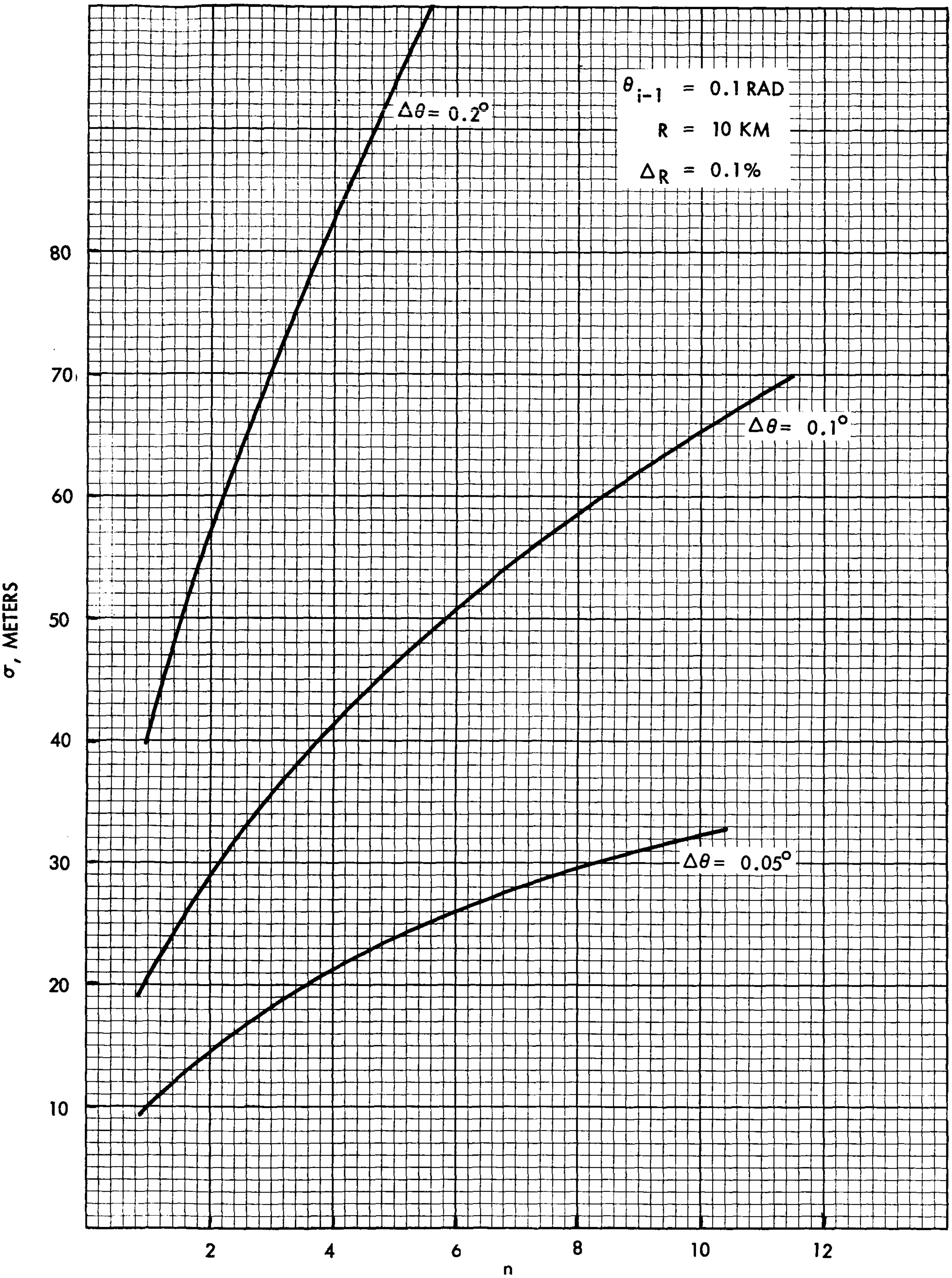


FIGURE II-1. ALTITUDE VARIANCE VERSUS NUMBER OF STEPS

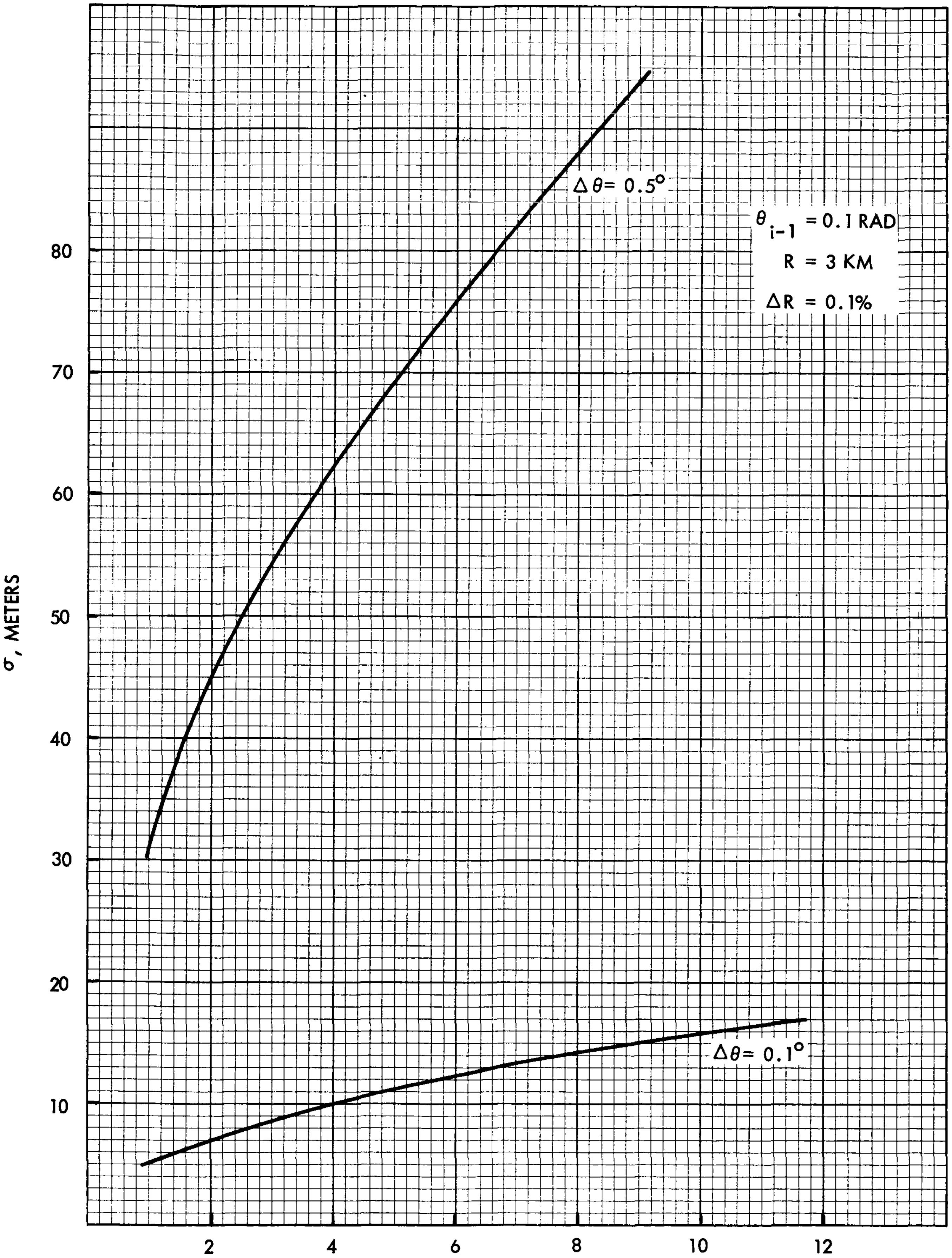


FIGURE II-2. ALTITUDE VARIANCE VERSUS NUMBER OF STEPS

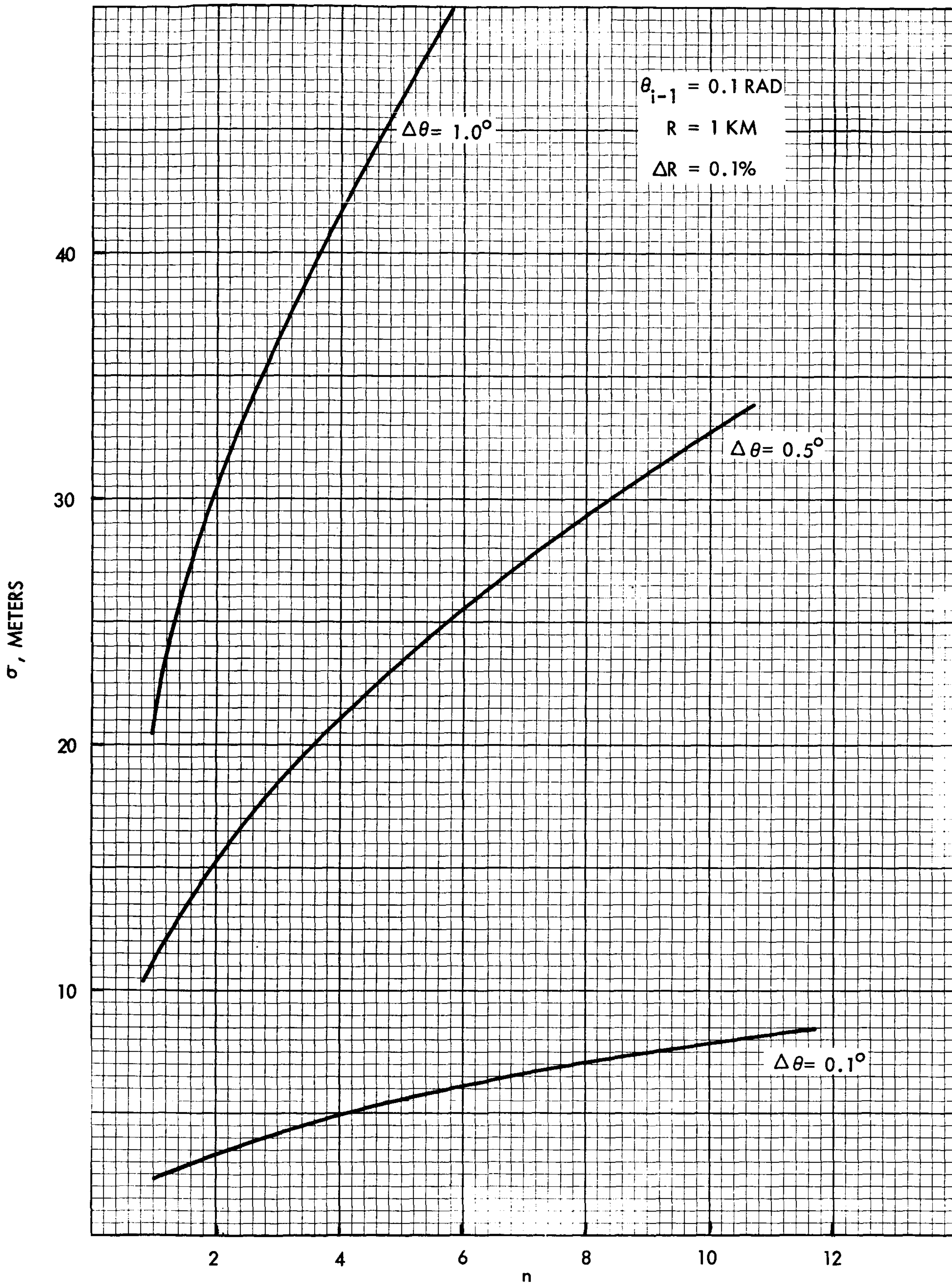


FIGURE II-3. ALTITUDE VARIANCE VERSUS NUMBER OF STEPS

APPENDIX C

GRAVITY GRADIOMETER CHARACTERISTICS

The very complete summary of gravity gradiometer characteristics shown below, although not directly pertinent to the elevation determination problem, is included for possible scientific interest. The Tables were obtained from Reference 12 and are self-explanatory.

GRADIOMETER CHARACTERISTICS

GRADIOMETER CHARACTERISTIC	GRADIOMETER TYPE		
	Vibrating String	Rotating Gradiometer	Electrostatic Accelerometers
1. Power			
a) Instruments	0.5 microwatts constant	0	0.1 watts
b) Electronics (signal processing and amplification, temperature control, power supply, and other control functions)	5 watts constant	8 watts constant	8.9 watts average 9.8 watts peak
c) Heaters	2.5 watts	2 watts average (heater power assumes passive spacecraft thermal control to $\pm 15°C$)	12 watts average 22 watts peak Heater power assumes passive spacecraft thermal control to be $\pm 15°C$
d) Total Power	7.5 watts average per instrument	10 watts average; 12 watts peak	21 watts average 31.8 watts peak
2. Weight			
a) Instruments	12 pounds	8 pounds	3 pounds
b) Electronics	3 pounds	3 pounds (electronics and heater)	7 pounds
c) Heaters	0.5 pound	---	1 pound
d) Accelerometer Interconnect Structure		---	1.6 pound (assumes one foot separation between accelerometers)
e) Total Weight	15.5 pounds	11 pounds	12.6 pounds
3. Volume & Form Factor			
a) Instruments	5.5 inches diameter x 12 inches long	14 inches diameter x 4 inches high	4-1/4 inch diameter cylinder, 18 inches long (pair of accelerometers and interconnecting tube). Connectors at side of accelerometer housing.
b) Electronics	2 inches x 4 inches x 5 inches	14 inches diameter x 2 inches high	4.5 inches x 5.5 inches x 8 inches
4. Threshold			
a) Analytical	1×10^{-11} g/foot (.3 EU)	0.05 EU for 100 second integration time determined by thermal noise limit	10^{-11} g
b) Demonstrated	MIT, evaluating a 30 g vibrating string accelerometer, has shown better than 10^{-8} g sensitivity.	600 EU under operating conditions (rotating in Earth environment); 0.2 EU under calibrated conditions (stationary with rotating test masses)	10^{-8} g

GRADIOMETER CHARACTERISTICS (Continued)

GRADIOMETER CHARACTERISTIC	GRADIOMETER TYPE		
	Vibrating String	Rotating Gradiometer	Electrostatic Accelerometers
9. Instrument Transfer Function (equation predicting instrument output for any defined input)	Standard underdamped second order mass-spring system; natural frequency about 15 cps	For a single transducer torsional type gradiometer the output voltage is given by: $V = \frac{3Qc\sigma g}{\lambda \omega_n^2} \frac{GM}{R^3}$ where: Q is the quality factor of the sensor head; c is the thickness of the flexural pivot spring; λ is the length of the flexural pivot spring; σg is the transducer gauge factor; ω_n is the resonant frequency of the sensor head; $\frac{GM}{R^3}$ is the gravitational gradient input	No data available.
10. State of Development	Vibrating string accelerometer is a fully developed and qualified sensor with a dynamic range of over 10^7. More than 1000 have been produced and it has been used in missile guidance systems. Gradiometer has not yet been developed.	Proof of principle models have been tested in laboratory with 1 EU measured.	Miniature electrostatic accelerometer under development since 1959. Several flights with accelerometer calibrated at 10^{-8} g.
11. Predicted Gradiometer Development Cost (order of magnitude) a) 1969 Flight b) 1970 Flight	 <$2,500,000 <$2,000,000	No data available.	 $2,000,000 $1,500,000
12. Launch Environment Capability	Requirements of all standard vehicles can be met. Gradiometer is automatically caged when not in orbit.	Can readily accommodate typical launch dynamic environments.	Equivalent to Thor-Delta & Atlas-Agena environments

GRADIOMETER CHARACTERISTICS (Continued)

GRADIOMETER CHARACTERISTIC	GRADIOMETER TYPE		
	Vibrating String	Rotating Gradiometer	Electrostatic Accelerometers
5. Bias			
a) DC Acceleration	10,000 to 1 rejection	Insensitive	10^{-5} g/cross-axis g capability
b) AC Acceleration	Insensitive	10,000 to 1 rejection @ 2 x rotation frequency; insensitive to other frequencies	Insensitive
c) DC Angular Velocity	$\frac{1\ \text{EU}}{4.5\ \text{deg./hr.}}$	$\frac{1\text{EU}}{4.5\ \text{deg./hr.}}$	$\frac{1\ \text{EU}}{\frac{1\ \text{EU}}{4.5\ \text{deg./hr.}}}$
d) AC Angular Velocity	Insensitive	1 EU/10^{-7} Ω^2 @ 2 x rotation frequency; insensitive to other frequencies	Same
e) Ambient Temperature Variation	100 EU/°C (internal)	Insensitive	10^{-7} g/°F/cross-axis g capability
f) Drift with Time	1 EU/hour	—	—
6. Scale Factor Error			
a) DC Acceleration	Insensitive	Insensitive	Insensitive
b) AC Acceleration	Insensitive	Insensitive	Insensitive
c) DC Angular Velocity	Insensitive	Insensitive except about spin axis (0.1% rotation speed drift will cause 1.0% scale factor change for a sensor having a Q of 100 and integration time of 10 seconds)	Insensitive
d) AC Angular Velocity	Insensitive	Insensitive	Insensitive
e) Ambient Temperature Variation	Insensitive due to local thermal control	0.5%/°C	.01%/°F
7. Scale Factor Drift with Time	10^{-5}% per hour	None in sensor; possible small value in electronics	.002%/month (turn-on to turn-on repeatability 0.03%)
8. Sensor Integration Time	10 seconds (continuous readings)	10 seconds (continuous readings)	Function of scale factor, required dynamic range, and allowable error. For lunar missions (1 EU resolution and 10^4 dynamic range), a 20 second integration time yields 5% error.

GRADIOMETER REQUIREMENTS

GRADIOMETER REQUIREMENT	GRADIOMETER TYPE		
	Vibrating String	Rotating Gradiometer	Electrostatic Accelerometers
1. Voltage Input	28 VDC ± 10%	28 VDC ± 10%	28 VDC ± 10%
2. Temperature Control (self-contained in gradiometer system) a) Instrument b) Electronics c) Interconnecting Structure	(Passive capability of spacecraft assumed to be ±15°C) ±0.01°C ±0.5°C —	(Passive capability of spacecraft assumed to be ±15°C) ±10°C (control) ±1°C (knowledge)	(Passive capability of spacecraft assumed to be +15°C) ±0.1°C ±0.05°C ±0.1°C
3. Input Signals	a) Uncaging of instrument b) Readout command c) Clock input	a) Readout commands b) Clock input	a) Switching of electrostatic support voltage level b) Readout command c) Clock input d) Mode switching (to vary integration time)
4. Output a) Digital	a) 16 bit gravity gradient word b) Answer from readout command (10 second readout interval)	a) 14 bit gravity gradient word b) Answer from readout command (10 second interval)	a) Two lines (+ or – "g") each having pulse rate proportional to "g" level with a full-scale value of 5000 pps. Instrument accuracy limit -0.01%. b) Answer from readout command
b) Analog	a) Gradiometer temperature (2 points) b) Electronics temperature (10 minute sampling interval)	a) Sensor temperature (±1°C)	a) Accelerometer temperature (2) b) Electronics temperature (2) (10 minute sampling interval; 5% accuracy) c) Pickoff voltage (2) d) Suspension voltage (2) (1 second sampling interval; 0.01% accuracy) DC to 10 cps frequency response
5. Sensor Orientation	a) Inertially fixed b) Aligned with local vertical if nominal 2000 EU bias is acceptable	Rotation axis normal to orbital plane preferred. (Orbit regression will gradually separate sensitive axis from orbital plane.) Other orientations acceptable.	a) Inertially fixed b) Aligned with local vertical if nominal 2000 EU bias is acceptable.

GRADIOMETER REQUIREMENTS (Continued)

GRADIOMETER REQUIREMENTS	GRADIOMETER TYPE		
	Vibrating String	Rotating Gradiometer	Electrostatic Accelerometer
6. Attitude Control Accuracy	Requirement determined by nature of experiment	Not critical; knowledge to ±1° desired	
7. Attitude Rate Control	1.4 degrees/hour (for 0.1 EU bias) 4.5 degrees/hour (for 1.0 EU bias) 10 degrees/hour (for 5 EU bias)	About axis normal to spin vector 1.4 degrees/hour (0.1 EU bias) 4.5 degrees/hour (1.0 EU bias) 10 degrees/hour (5 EU bias)	1.4 degrees/hour (for 0.1 EU bias) 4.5 degrees/hour (for 1.0 EU bias) 10 degrees/hour (for 5 EU bias)
8. Rate Sensing a) At ω = 1.4°/hour	0.56 degrees/hour (0.1 EU uncertainty) 2.5 degrees/hour (0.7 EU uncertainty)	0.56 degrees/hour (0.1 EU uncertainty) 2.5 degrees/hour (0.7 EU uncertainty)	0.56 degrees/hour (for 0.1 EU uncertainty) 2.5 degrees/hour (for 0.7 EU uncertainty)
b) At ω = 4.5°/hour	0.22 degrees/hour (0.1 EU uncertainty) 1.35 degrees/hour (0.7 EU uncertainty) 0.1 degrees/hour (0.1 EU uncertainty)	0.22 degrees/hour (0.1 EU uncertainty) 1.35 degrees/hour (0.7 EU uncertainty)	0.22 degrees/hour (for 0.1 EU uncertainty) 1.35 degrees/hour (for 0.1 EU uncertainty)
c) At ω = 10°/hour	0.7 degrees/hour (0.7 EU uncertainty)	0.1 degrees/hour (0.1 EU uncertainty) 0.7 degrees/hour (0.7 EU uncertainty)	0.1 degrees/hour (for 0.1 EU uncertainty) 0.7 degrees/hour (for 0.7 EU uncertainty)
9. Instrument Alignment Accuracy	±6 minutes of arc	Not critical if misalignment is constant thereby producing constant bias.	±1 minute of arc
10. Instrument Calibration	The weak cross axis suspension of the gradiometer precludes calibration on the ground. It is presumed that orbital calibration would involve positioning a test mass at precisely known distances from the gradiometer and at various angles to the sensitive axis. Since the system will be external to the command module, this requires remote positioning of the test mass. It also precludes any adjustment.	Full calibration is possible on the ground at same scale factor as will be used in space. Verification calibration in orbit would involve positioning at test mass at precisely known distances from the system and at various angles to the spin vector. Since the system will be external to the command module, this requires remote positioning of the test mass.	Scale factor and scale factor linearity are calibrated at higher gradient levels on the ground and extrapolated to the low range anticipated in orbit. Mass attraction calibration of scale factor and null bias is performed in orbit. The procedure involves positioning a test mass at precisely known distances from the accelerometer array and at various angles to the sensitive axis. This must be done with the system external to the Apollo vehicle and thus requires remote positioning of the test mass. It also precludes any instrument adjustment.
11. Spin Requirements a) Speed	N/A	300 rpm desired for Q = 100. Lower speed reduces Q approximately proportionately.	N/A
b) Speed control	N/A	±1%	N/A
c) Speed knowledge	N/A	±0.1%	N/A

POSSIBLE GRADIOMETER PROBLEMS AREAS

Vibrating String	Rotating Gradiometer	Electrostatic Accelerometers
1. The inability to calibrate the system on the ground implies a commitment to launch without a calibrated instrument. 2. No data is available on instrument sensitivity to thermal noise. Since this has presented a problem in other gradiometers, it should be considered for the vibrating string system. 3. Temperature control may be critical, particularly in a spacecraft whose internal excursions may vary as much as $\pm 15°C$.	1. Misalignment of the spin vector relative to spacecraft (and sensor) geometry may occur due to initial spin-up misalignment, inertia unbalance, etc. As a result, a steady state bias may be produced in gradiometry data. Although the spin vector will gradually deviate from normal to the orbital plane as a result of external torques and orbital regression, this deviation is not regarded as significant, and should not require repointing of the spin vector. 2. Nutation of the spinning spacecraft will introduce corss-axis ratios into the gradiometer. These rates must either be confidently below the 0.5 EU level (4.5 degrees per hour) or must be measurable to considerable accuracy.	1. If calibration at orbital sensitivity level is not possible on the ground, this capability must be provided in flight. 2. Thermal noise limits may require increasing the proof mass. The effect on instrument performance must be evaluated. 3. Temperature control may be critical. A typical MESA scale factor coefficient is given as 0.2% per °F which gives an acceptable amount of scale factor shift with temperature control to within $\pm 0.2°F$. However, this typical value may not apply to an instrument of several orders of magnitude greater sensitivity. Critics of the MESA technique have suggested that the scale factor is influenced at the rate of 10^{-6} g/°C and hence requires much finer temperature control. 4. Since the accelerometers do not measure gravity gradient directly, the characteristics of the two instruments must be matched with considerable precision. The feasibility of such matching may be in question. 5. In order to reduce sampling time to reasonable levels, automatic scale ranging may be required. However, the feasibility of ranging is contingent upon maintaining the required resolution of approximately 1.5×10^{-11} g.

APPENDIX D

DATA LINK CALCULATIONS

To obtain an estimate of the accuracy with which h can be measured, the following link calculations are needed. Assume:

Prime power = 20 watts

Transmitter power = 10 watts

Antenna gain (both ends) = 3 db (assuming the 2-element array)

Frequency = 1 Ghz

Distance = 100 n.mi. (assumed maximum tracking range)

The path loss between omniantennas is

$$L = 38.1 + 20 \log f + 20 \log D$$

$$= 38.1 + 60 + 40 = -138 \text{ db}$$

Transmitter power = 10 dbw

Antenna gain = 3 db

ERP = 13 dbw

Received power at satellite antenna = -138 + 13 = -125 dbw

Satellite antenna gain = 3 db

Received power at satellite = -122 dbw.

To compute noise, assume a 50 Hz phase lock loop bandwidth. Then,

Noise = KTB where

K = Boltzman constant = -228.6 dbw

T = receiver effective temperature = $1000^{\circ}K$ = 30 db

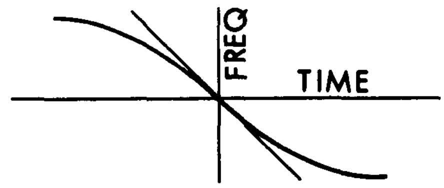
B = bandwidth = 50 Hz = 17 db

Noise = -181.6 dbw

Carrier power = -122.1 dbw

59.5 db = S/N in satellite.

If 3 db degradation is assumed for down transmission and a 6.5 db margin, the final S/N at the Rover is 50 db. The receiver frequency as a function of time will have a curve of the form shown, where the slope through the origin equals



$$\frac{f_o v^2}{ch} .$$

The single sample accuracy of a 50 db S/N at the point of maximum slope is 50 db $\sim$ 100,000:1 in power or 316:1 in amplitude.

REFERENCES

1. Lunar Navigation Study-Final Report, June 1965. Bendix Systems Division of Bendix Aviation Corp., Contract NAS8-11292, Ann Arbor, Michigan.

2. R. G. Brereton, "Lunar Mission Planning", JPL Space Programs Summary, Vol. III, pgs. 37-54.

3. R. G. Brereton, R. B. Coryell, E. A. Howard, D. K. Rubin, "Experiments in Remote Geological Reconnaisance and Landmark Identification", JPL 760-41, October 1969.

4. R. G. Brereton, "Automated Post-Missions", JPL Internal Document, Feb., 1970.

5. R. B. Coryell, "Lunar Landmark Study", JPL 760-45.

6. R. B. Coryell, D. K. Rubin, "Experiments in Piloting by Landmark on the Moon", JPL 360-37, April 1970.

7. P. H. Savet, "Self-Contained Method of Lunar Navigation", Grumman Res. Dept. Memo RM-463, Dec. 1969, Bethpage, N.Y.

8. P. H. Savet, et al, "Gravity Gradiometry", Grumman Res. Dept. Report RE-313, December 1967.

9. C. C. Bell, "Lunar Orbiter Selenodesy Feasibility Demonstration", Hughes Research Lab., January 1970.

10. S. Hansen, "A Highly Stable Geophysical Tiltmeter", Hughes Research Labs., December 1968.

11. "Lunar Orbiter Selenodesy Sensor Study", Hughes Research Labs., March 1970, Proposal No. 70M-0265/B8937-2.

12. "Gravity Gradiometry-Mission Feasibility Study Final Report", Noltec Corp., Rockville, Maryland, Contract No. NAS 8-21182.

13. C. W. B. McCormick, "Trip Report; U.S. Geological Survey, Flagstaff, May 1969", JPL Interoffice Memo, 317-136.

14. J. C. Harrison, "The Measurement of Gravity", Proceedings of IRE, Nov., 1962.

15. P. M. Muller, W. L. Sjogren, "Mascons: Lunar Mass Concentrations", Science 161, pgs. 680-684, 1968.

16. P. M. Muller, W. L. Sjogren, "Information From Deep-Space Tracking", Physics Today, Vol. 22, No. 7.

17. P. M. Muller, W. L. Sjogren, "Lunar Gravity and Mascons", Applied Mechanics Reviews, pgs. 955-959, 1968.

18. P. M. Muller, W. L. Sjogren, W. R. Wallenhaupt, "Lunar Gravity Over Large Craters From Apollo 12 Tracking Data", Science 168, pgs. 477-479, April 1970.

19. Science, Vol. 162, pgs. 1402-1410. A series of several articles discussing various aspects of the mascon problem.

20. "Laser Altimeter for Lunar Orbit Metric Photography Experiment", November 1969, Hughes Proposal to NASA.

21. D. B. James, "A Method for Navigating on and Near the Moon", Bellcomm, Inc., June 1968. (This reference was provided by D. Rubin. It involves differential ranging between two stations on the moon and two or more station on earth.)

22. A. T. Owens, "Design on a Phase Locked Loop for Doppler Tracking", Hughes IDC 2230.5/62, June 1962.

23. J. R. Vyce, "An Introduction to Gyro Optical Pick-offs", Journal of Spacecraft and Rockets, pgs. 681-688, 1965.

24. Bendix and Gramman Reports, "Dual Mode Lunar Roving Vehicle - Design Definition Study", 1969-70.

25. R. B. Odden, T. T. Trexler, "Technology for Lunar Surface Navigation", Bendix Technical Journal, Vol. 1, pgs. 44-52, 1968.

26. A. G. Webster, "The Dynamics of Particles", Second Edition, Dover Publications, New York.

27. M. R. Macomber, M. Fernandez, Inertial Guidance Engineering, Prentice-Hall, Inc., 1962, pgs. 371-398.